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ICMME²⁰²⁵

INTERNATIONAL CONFERENCE ON MATHEMATICS
AND MATHEMATICS EDUCATION

MATHEMATICS
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COMPUTER SCIENCE
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ABSTRACT BOOK

Mathematics in Istanbul, Bridge Between Continents

SEPTEMBER 11-13, 2025



ISTANBUL MEDENIYET UNIVERSITY

Istanbul/Türkiye

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*International Conference on Mathematics and
Mathematics Education (ICMME-2025), İstanbul Medeniyet
University, İstanbul, Türkiye, September 11-13, 2025*
"Mathematics in İstanbul, Bridge Between Continents"



International Conference on Mathematics and Mathematics Education
(ICMME - 2025)

İstanbul Medeniyet University, İstanbul, Turkey, 11-13 September 2025

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PREFACE

The International Conference on Mathematics and Mathematics Education (ICMME-2025) was held in İstanbul, Türkiye, from September 11-13, 2025. The conference brought together researchers, educators, and practitioners to exchange ideas, present recent research findings, and discuss future challenges in mathematics and related disciplines. The scientific program included invited plenary lectures and short oral presentations, providing a comprehensive overview of current developments in the field.

MATDER – Mathematicians Association was established in 1995 by mathematicians in Türkiye. Since its foundation, MATDER has organized 14 national and 7 international mathematics symposia. Over the years, these meetings have become major scientific events at both national and international levels. Covering a wide range of topics in pure and applied mathematics, mathematics education, engineering mathematics, and computer science, the ICMME conferences have consistently attracted significant interest from mathematicians and engineers in academia.

The previous ICMME conferences were held in Nevşehir (ICMME-2024), Denizli (ICMME-2022), Ankara (ICMME-2021), Konya (ICMME-2019), Ordu (ICMME-2018), Şanlıurfa (ICMME-2017), and Elazığ (ICMME-2016). This year, ICMME-2025 was hosted by İstanbul Medeniyet University, İstanbul, Türkiye, September 11–13, 2025, as an international scientific conference.

The main objective of ICMME-2025 is to contribute to the advancement of mathematical sciences, mathematics education, computer sciences, and their applications, as well as to provide a platform for international collaboration among researchers, educators, mathematicians, statisticians, and interdisciplinary scholars from around the world.

Abstracts of all accepted presentations are included in these conference proceedings. In addition, selected and peer-reviewed full papers will be published in the following journals:

- [Advanced Studies: Euro-Tbilisi Mathematical Journal](#)
- [Turkish Journal of Mathematics and Computer Science \(TJMCS\)](#)
- [International Journal of Maps in Mathematics](#)
- [Selcuk University Journal of Engineering Sciences](#)
- [Gazi Eğitim Bilimleri Dergisi](#)
- [Advances in Differential Equations and Control Processes](#)

The ICMME-2025 conference is jointly organized by MATDER – The Mathematicians' Association and İstanbul Medeniyet University. We sincerely thank all participants, authors, reviewers, and invited speakers for their valuable contributions to the success of this conference.

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INVITED SPEAKERS



ARTIFICIAL INTELLIGENCE: A NEW BRIDGE BETWEEN DISCIPLINES, LANGUAGES, AND CONTINENTS

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ABSTRACT

Artificial Intelligence (AI) has emerged as a transformative technology that transcends traditional boundaries in our increasingly interconnected world. This presentation examines how AI creates new bridges between diverse disciplines, languages, and geographical regions. In today's landscape of technological advancement, AI systems act as powerful mediators capable of synthesizing methodologies from disparate fields—from computer science and mathematics to linguistics, psychology, and ethics. We will explore how Generative Artificial Intelligence (Generative AI) and Large Language Models (LLMs) serve as universal translators and knowledge integrators, connecting previously isolated domains of human activity. The presentation will clarify the fundamental architecture of these systems, including tokenization processes, parameter functions, and the underlying logic that enables these models to generate coherent, contextually appropriate content. Beyond technical aspects, we will analyze how AI bridges linguistic barriers, providing unprecedented access to information regardless of one's native language. Drawing a parallel to İstanbul's role in connecting Europe and Asia, we will demonstrate how AI links distant fields of knowledge, allowing cross-pollination of insights between disciplines. Through real-world applications, we will showcase how these systems facilitate cross-disciplinary collaboration and global knowledge exchange. By understanding both the capabilities and limitations of these powerful tools, we can better navigate the ethical considerations they present while harnessing their potential. As we stand at this technological crossroads, AI offers a unique opportunity to build new bridges of understanding in our global community.



RENORMALIZATION-GROUP THEORY: PHASE TRANSITIONS, SPIN-GLASS CHAOS, MUSIC, BRAIN, HIGH SCHOOL AND UNDERGRADUATE STUDENT RESEARCH EMPOWERMENT

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ABSTRACT

Renormalization-group theory has become a widely and most easily used theoretical method for the statistical mechanics of complex systems.[1] For example, a complexity classification scheme is developed from the fractal spectra of spin-glass chaos and demonstrated with multigeographic multicultural music and brain electroencephalogram signals.[2] Systematic patterns are found to emerge. Chaos under scale change is the essence of spin-glass ordering and can be obtained, continuously tailor-made, from the exact renormalization-group solution of Ising models on frustrated hierarchical lattices. The studied music pieces are from genres of Turkish music, namely Arabesque, Rap, Pop, Classical, and genres of Western music, namely Blues, Jazz, Pop, Classical. A surprising group defection occurs. A multitude of phases and phase diagrams can be calculated for complex spin systems including frozen disorder.[3] Equilibrium and non-equilibrium (spinodal) phase transitions are obtained.[4] Renormalization-group theory provides the opportunity for undergraduate [2,3] and high school [5] students to accomplish research published in the top journals.

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ON WEIGHTED ESTIMATES FOR OPERATORS OF HARMONIC ANALYSIS IN MORREY SPACES VIA POINT-WISE ESTIMATES FOR WEIGHTED HARDY OPERATORS

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ABSTRACT

In this lecture I focus on results for weighted boundedness in Morrey spaces, I obtained both decades ago and recently, for Hardy, potential, maximal and singular operators, mainly concentrating on methods in obtaining these results. One of the main features of these results is the possibility to get weighted estimates "beyond the Muckenhoupt range". As a crucial role of our approach, we emphasize the importance of our point-wise estimates for weighted Hardy operators via the norm of the domain Morrey space. These estimates allow in fact to have a result on boundedness of weighted Hardy operators from a Morrey space into any Banach function space with the lattice property. This holds for general weights including the two-weight case. For other operators, i.e. potential, maximal and singular operators, we deal with certain classes of radial weights. Properties of weights in these classes allow to point-wisely reduce the boundedness of the weighted operators under consideration to the boundedness of the corresponding non-weighted operator and weighted Hardy operators and, in the case of maximal and singular operators, also certain hybrids of Hardy and potential operators. The study of the boundedness of these hybrids in Morrey spaces leads to the final result on the weighted boundedness of maximal and singular operators.



GENERALIZED WEIGHTED LORENTZ SPACES

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ABSTRACT

We give a new characterization of a continuous embedding between two function spaces of type GF . Such spaces are governed by functionals of type

$$\|f\|_{GF(r,q;w,\delta)} := \left(\int_0^L \left(\frac{1}{\Delta(t)} \int_0^t f^*(s)^r \delta(s) ds \right)^{\frac{q}{r}} w(t) dt \right)^{\frac{1}{q}},$$

in which f^* is the nonincreasing rearrangement of f , $r, q \in (0, \infty)$, w, δ are weights and $\Delta(t) = \int_0^t \delta(s) ds$ for $t \in (0, \infty)$. To characterize the embedding of such a space, say $GF(r_1, q_1; w_1, \delta_1)$, into another, $GF(r_2, q_2; w_2, \delta_2)$, means to find a balance condition on the four positive real parameters and the four weights in order that an appropriate inequality holds for every admissible function. We develop a new discretization technique that will enable us to get rid of restrictions on parameters imposed in earlier work, such as the non-degeneracy conditions or certain relations between the r 's and q 's. Such restrictions were caused mainly by the use of duality techniques, which we avoid in this paper. On the other hand, we consider here only the case when $q_1 \leq q_2$ to keep the paper in a reasonable length.

Key Words: Lorentz space, weighted inequality, integral operator.

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PROBLEM ÇÖZMEDE YÖNELİMLER: SOSYAL ADALET VE DEĞER TEMELLİ PROBLEM ÇÖZME YAKLAŞIMLARI

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ABSTRACT

Matematik, çoğu zaman soyut bir alan gibi görülse de aslında hayatın her alanına dokunan, bireylerin ve toplumların daha adil, daha eşitlikçi bir dünya kurmalarına katkı sağlayabilecek güçlü bir araçtır. Son yıllarda matematik eğitiminde öne çıkan yönelimlerden biri, **sosyal adalet temelli problem çözme yaklaşımlarıdır**. Bu yaklaşım, öğrencilerin yalnızca doğru cevaba ulaşmalarını değil, aynı zamanda matematiksel akıl yürütmeyi toplumsal sorunları anlamada, sorgulamada ve çözüm üretmede bir araç olarak kullanmalarını hedefler. Çevresel adalet, toplumsal eşitsizlikler, fırsat eşitliği gibi konular matematiksel problemlere yansıtıldığında; öğrenciler, matematiğin yaşamın her alanında işlevsel ve dönüştürücü gücünü deneyimleme fırsatı bulurlar. Bununla birlikte, **değer temelli problem çözme yaklaşımları**, öğrencilerin problem çözme süreçlerinde işbirliği, empati, sorumluluk, dürüstlük ve saygı gibi evrensel değerleri deneyimlemelerini sağlar. Bu sayede matematik eğitimi, sadece akademik başarıya indirgenmeyip bireyin karakter gelişimine, etik karar verme becerilerine ve toplumsal sorumluluk bilincine de katkı sunar. Sosyal adalet ve değer temelli problem çözme yaklaşımları, bizlere matematiği yalnızca öğretilcek bir ders olmaktan çıkarıp, yaşamın kendisiyle bütünleşen bir öğrenme alanı haline getirme imkânı tanımaktadır. Sosyal adalet ve değer temelli problemler, öğrencilerin yalnızca matematiksel becerilerini değil aynı zamanda toplumsal duyarlılıklarını, eleştirel düşünme ve karar verme yetilerini de geliştirmeyi amaçlamaktadır. Matematik eğitiminde sosyal adalet ve değer temelli problem çözme, öğrencileri hem akademik hem de toplumsal yaşamda daha bilinçli bireyler olmaya hazırlayan çağdaş bir eğilim olarak sunulmaktadır. Geleneksel matematik öğretiminde problem çözme çoğunlukla rutin işlemler ve doğru cevaba ulaşma odaklı iken, günümüzde problem çözme süreci sosyal, kültürel ve değerler boyutlarıyla yeniden ele alınmaktadır.



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Matematik eğitiminde sosyal adalet ve değer temelli bakış açısını bütünleştirmek, biz eğitimciler için bir sorumluluk olduğu kadar bir fırsattır. Çünkü bu yaklaşım, hem öğrenme motivasyonunu artırmakta hem de öğrencilerin eleştirel düşünme ve problem çözme becerilerini daha anlamlı bir bağlamda geliştirmelerine olanak tanımaktadır.



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INTERPOLATION THEOREMS FOR NONLINEAR OPERATORS IN GENERAL MORREY-TYPE SPACES

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ABSTRACT

The talk will start with a survey of results on interpolation results for quasi-linear operators in Morrey and Morrey-type spaces. The main part of the talk will be devoted to interpolation of a wide class of nonlinear operators, including nonlinear Urysohn operators, in general local Morrey-type spaces and their generalizations. The results which will be presented are based on joint research with E. Nursultanov, D. Chigambaeva (Darbaeva), A. Kalidoldai.



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ABSTRACTS OF ORAL PRESENTATIONS



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ALGEBRA AND NUMBER THEORY



New Extensions of Weakly 1-Absorbing Ideals in the Context of m-Systems

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ABSTRACT

In this talk, we will explore the notion of weakly S-1-absorbing prime ideals within the framework of noncommutative ring theory. This concept extends both weakly 1-absorbing ideals [1] and S-weakly prime ideals [2]. Given a ring R , a proper ideal P that is disjoint from a designated m -system S is defined to be weakly S-1-absorbing prime if, for any non-invertible elements k, m, l of R such that the product $kRmRl$ is nonzero and contained in P , there exists an element s of S for which either $km(s) \subseteq P$ or $l(s) \subseteq P$ holds.

Throughout the talk, we will present various structural results and characterizations of such ideals, with a focus on their behavior in specific classes of rings, such as prime rings, local rings, and rings that are simultaneously local and prime. One of the notable results includes a characterization theorem for weakly S-1-absorbing prime ideals in prime rings, offering necessary and sufficient conditions for their existence.

Key Words: noncommutative rings, S-weakly prime ideals, 1-absorbing prime ideals.

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The Norms of Some Circulant Matrices Associated with Leonardo Polynomials

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ABSTRACT

Each term of the Leonardo polynomials is related to the magical Fibonacci numbers, which whisper with nature's golden ratio. The Leonardo polynomials is defined by the following recurrence relation for $n \geq 2$

$$Le_n(x) = xLe_{n-1}(x) + Le_{n-2}(x) + x$$

with the initial conditions $Le_0(x) = 1, Le_1(x) = 2x - 1$. There is a relation between Leonardo and Fibonacci polynomials for $n \geq 0$ such that $Le_n(x) = 2F_{n+1}(x) - 1$.

An $n \times n$ square matrix C is called a circulant matrix if it is defined as follows:

$$C = \begin{pmatrix} c_0 & c_1 & c_2 & \cdots & c_{n-2} & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \cdots & c_{n-3} & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \cdots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \cdots & c_{n-1} & c_0 \end{pmatrix}.$$

The matrix C is determined to its first-row elements, other rows obtained by making each element circulate. Circulant matrices have many special types which are symmetric circulant, semi-circulant, skew circulant, r-circulant, g-circulant and geometric circulant matrices.

In our study, Leonardo polynomials and Fibonacci polynomials are introduced, generating functions and various sum formulas are proven. By utilizing the product of the generating function, the norms of some circulant matrices associated with Leonardo polynomials and Fibonacci polynomials are proven completely.

Key Words: Leonardo polynomials, circulant matrices, norms.



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A Note on Generalized Derivations in Prime Ring Satisfying an Identity on Lie Ideals

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ABSTRACT

Let $\mathcal{R}$ be a prime ring with center $Z(\mathcal{R})$, right Martindale quotient ring Q and extended centroid $\mathcal{C}$. By a derivation we mean an additive mapping $d: \mathcal{R} \rightarrow \mathcal{R}$ such that $d(xy) = d(x)y + xd(y)$ for all $x, y \in \mathcal{R}$. By a generalized derivation we mean an additive mapping $F: \mathcal{R} \rightarrow \mathcal{R}$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in \mathcal{R}$ where d is a derivation of $\mathcal{R}$ and d is called an associated derivation of F . An additive subgroup U of R is said to be a Lie ideal of R if $[U, R] \subseteq U$. In this work we will present, we aim to investigate an identity concerning about generalized derivations. More precisely, we prove the following:

If $\mathcal{R}$ admits generalized derivations F associated with a derivation d and G such that $[F(u), d(v)] = G([u, v])$ for all u, v in a non-central Lie ideal of $\mathcal{R}$ then $G = 0$ and one of the following holds:

- (i) there exists $p \in \mathcal{R}$ such that $F(x) = px$ for all $x \in \mathcal{R}$;
- (ii) $\mathcal{R} = M_2(\mathcal{C})$ and there exist $a, b \in \mathcal{R}$ such that $F(x) = ax + xb$ for all $x \in \mathcal{R}$ and $a = \sum_{i,j=1}^2 a_{ij}e_{ij}$, $b = \sum_{i,j=1}^2 b_{ij}e_{ij}$, where e_{ij} denotes the usual matrix unit with 1 in (i,j) -entry and zero elsewhere and $a_{ii} + b_{jj} = 0$ and $a_{ij} - b_{ij} = 0$ for all $1 \leq i \neq j \leq 2$.

Key Words: Prime ring, generalized derivations.

Mathematics Subject Classification: 16W25, 16N60.

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Structure and Interactions of Augmentation Submodules in G -set Modules

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ABSTRACT

In this study, we systematically investigate the structure and interactions of generalized augmentation submodules on the G -set module MS , constructed from a G -set S and a left R -module M . We begin by defining a suitable analogue of the classical augmentation ideal $\Delta_G(R)$ in the context of MS , introducing the notion of the relative augmentation submodule $\Delta_{G,H}(MS)$. The algebraic behaviour of these submodules is analysed in detail, with particular emphasis on their additive, intersectional, and conjugation properties, as well as their relationship to dimension subgroups and colon ideals.

First, we recall the classical correspondence between the augmentation ideal and the dimension subgroup in the group ring RG , and extend these concepts to the G -set module framework. Next, we develop fundamental theorems demonstrating the structural behaviour of $\Delta_{G,H}(MS)$ under algebraic operations. In particular, we prove that these submodules behave well under sums and intersections of normal subgroups, and we show that under a faithful action of G on MS , the intersection of relative augmentation submodules precisely recovers the intersection of the corresponding subgroups. Finally, we explore the behaviour of augmentation submodules and their associated filtrations under conjugation in two-sided G -sets. We demonstrate that the entire augmentation filtration $\{\Delta_{G,H}(MS)\}_{n \geq 1}$ is preserved under bi-set conjugation, leading to a graded RG -module isomorphism between conjugate G -set modules. These results provide new insights into the internal structure of G -set modules and extend the classical augmentation theory to a broader equivariant setting.

Key Words: Group ring, G -set module, augmentation module.



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From Quadratic Modules to Nil (2)-modules of Lie Algebras

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ABSTRACT

We known the notion of quadratic module of groups as an algebraic model for connected homotopy 3- types [1]. Commutative algebraic variation of a quadratic module is given by Arvasi and Ulualan in [3]. Lie algebra version of quadratic modules have been studied by Ulualan and Uslu in [4]. Using images of $M_{\alpha,\beta}$ functions, they also gave the relationship between simplicial Lie algebras and quadratic modules for Lie algebras. Also they have defined a functor from the category of 2- crossed modules to that of quadratic modules of Lie algebras. Then, from the category of braided crossed modules of Lie algebras a functor is defined to the quadratic modules of Lie algebras. The category of Nil (n)- modules equivalent to category of n - crossed modules. The forgetful functor from the category of quadratic modules to that of nil (2)- modules is a bifibration for group and lie algebras in the literature [5, 6].

In this study, following Baues, we give a functor from the category of quadratic modules to the category of nil (2)- modules.

Key Words: nil (2) modules, quadratic modules, crossed modules.

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SMOOTHNESS ANALYSIS OF SPECTRAL RADII BASED ON SOME WELL-KNOWN TOPOLOGICAL INDICES

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ABSTRACT

Topological indices, or graph invariants, are mathematical formulas based on various graph concepts such as vertex degrees, eigenvalues and distances within graphs. They are crucial tools employed in quantitative structure–activity relationship (QSPR/QSAR) models [1-2]. These models play an effective role in fields such as drug design processes and toxicology by predicting the physicochemical properties of molecular structures. The properties that a topological index must possess to be considered a good molecular structure descriptor applicable in QSAR/QSPR models have been debated for years. These debates were clarified with the acceptance of the thirteen properties presented by Milan Randić in [4] today. One of the most important and difficult to measure of the thirteen properties is “should change gradually with gradual change in structures”. To measure this property, an important and widely accepted method called “smoothness of a topological index” was proposed in [3]. In the same paper, the smoothness values of twelve vertex-degree-based topological indices were investigated, while eigenvalue-based topological indices were left for future studies.

This study focuses on the spectral radius as one of the eigenvalue-based topological index types. The smoothness values of the first Zagreb-spectral radius ($\lambda_1^{M_1}$), the second Zagreb-spectral radius ($\lambda_1^{M_2}$), the atom-bond connectivity-spectral radius (λ_1^{ABC}), the augmented Zagreb-spectral radius (λ_1^{AZI}), the symmetric division deg-spectral radius (λ_1^{SDD}), the sum-connectivity-spectral radius (λ_1^{SCI}), the geometric-arithmetic-spectral radius (λ_1^{GA}) and the harmonic-spectral radius (λ_1^H), which are some of the spectral radii based on well-known topological indices [5], have been calculated on trees to determine which one has the most optimal values.



Key Words: Topological indices, eigenvalue-based topological indices, smoothness of topological indices

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Quadratic Modules for Leibniz Algebras

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ABSTRACT

Leibniz algebras can be seen as a non-commutative generalization of Lie algebras. As an immediate consequence, every Lie algebras are Leibniz algebras. Crossed modules of Leibniz algebras were introduced Loday - Pirashvili and Casas in [1,2]. Later, the category of 2-crossed modules of Leibniz algebras and the natural equivalences between crossed squares and cat^2 Leibniz algebras were given [4].

We known the notion of quadratic module of groups as an algebraic model for connected homotopy 3 types [5]. Lie algebra version of quadratic modules have been studied by Ulualan and Uslu in [7]. They have defined a functor from the category of 2 crossed modules to that of quadratic modules of Lie algebras.

In this study we give the notion of quadratic module for Leibniz algebras and explore the connections between this structure, 2 crossed modules of Leibniz algebras and simplicial Leibniz algebras.

Key Words: Leibniz algebras, crossed modules, quadratic modules.

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Variations of Domination In Partially Ordered Sets

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ABSTRACT

A dominating set in a graph G is a subset D of its vertices such that every vertex of G either belongs to D or has a neighbour in D . The domination number of a graph G is defined as the minimum size of a dominating set in G . This parameter has been extensively studied in graph theory, with numerous variants and applications. In [1], the concept of domination is extended from graphs to the partially ordered sets (posets), thereby opening a new line of investigation in the interaction between poset theory and graph theory.

In this study, we introduce several new domination parameters for posets and examine their structural properties. In particular, we identify which of these parameters remain invariant under taking comparability graphs, leading to a class of comparability invariants. These invariants provide a useful bridge between the study of posets and the well-established theory of domination in graphs. Furthermore, we establish explicit connections between classical graph parameters (such as domination number, total domination number, vertex cover number, and biclique vertex partition number) and the domination parameters of the associated posets. By means of these correspondences, we aim to obtain reductions between problems in poset theory and graph theory, thereby enriching the tools available for analyzing complexity questions and structural properties in both settings.

Key Words: Partially ordered sets, posets, domination, graphs, comparability..

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Generalized Prime and Maximal Ideals and Topologies in Commutative Rings

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ABSTRACT

The study of prime and maximal spectra, with their associated Zariski topologies, forms a foundation of commutative algebra, providing a bridge to algebraic geometry. Within this framework, pm-rings, where every prime ideal is contained in a unique maximal ideal, have been extensively studied for their distinctive topological properties. Another important class is that of clean rings, where every element is a sum of a unit and an idempotent, with strong connections to pm-rings. Motivated by recent generalizations of prime and maximal spectra by using a multiplicatively closed subset S , this talk introduces the concept of S -pm-rings, a natural extension of pm-rings, and establishes fundamental characterizations linking their algebraic structure to the topology of their S -spectra. We establish the existence of a retraction from the S -prime spectrum onto the S -maximal spectrum, relating this to homotopy theory. Furthermore, we examine how the S -pm-ring property connects to the normality of the S -prime spectrum and the Hausdorff property of the S -maximal spectrum. Finally, we unify these ideas by exploring their relationship with S -clean rings, showing the connection between the S -clean property and the S -pm-ring condition. These results systematically extend classical theorems in commutative algebra and algebraic geometry to the generalized S -version context.

Key Words: pm-rings, Zariski topology, prime and maximal spectra, retraction, clean rings.

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New Perspectives on Generalized Prime Ideals

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ABSTRACT

This study examines a specific class of ideals within the framework of commutative ring theory. A generalized version of the classical prime ideal concept is introduced by utilizing a particular function defined on the set of ideals of a ring. The newly defined class is characterized by a condition involving products of elements belonging to a specific difference set of ideals, where certain powers of the elements are required to be elements of the ideal. Fundamental properties of this structure are investigated and supported by various examples. Furthermore, connections with significant ring-theoretic structures, such as von Neumann regular rings, are established, and their characterizations are presented. In the final part of the study, a novel approach is proposed for valuation domains, and special types of ideals are defined for such domains.

Key Words: Prime ideal, von Neumann regular ring, valuation domain.

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Formal Graphs

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ABSTRACT

Classical Cartesian graphs are usually obtained by viewing y as a function of a real variable x , where the function is defined by *substituting* real numbers for x inside a syntactic expression. This operational view is convenient but fragile: removable singularities (e.g., $y=(x^2-1)/(x-1)$ at $x=1$) and similar “indeterminate forms” arise from naïve substitution into arbitrary expressions. These indeterminate forms, such as $0/0$, traditionally require limits or calculus for resolution. This paper introduces **formal graphs**, a lightweight algebraic method using indeterminates and ring homomorphisms to handle such forms without invoking calculus. By treating expressions as elements of an algebraic structure (e.g., a polynomial ring $R[x]$, a fraction field $R(x)$, etc.) and graphing via homomorphisms, formal graphs provide a robust, exception-free visualization aligned with intuitive algebraic simplification. We illustrate the method on canonical examples, including rational functions and difference quotients, and discuss advantages over traditional variable-based graphing. The upshot: standard calculus facts (e.g. the derivative as the simplified difference quotient at $h = 0$) fall out algebraically.

We present the approach at three levels of sophistication. **Graduate level.** We construct $R(x)$ abstractly as $\text{Frac}(R[x])$ with x an indeterminate, and define a family of ring homomorphisms $\text{ev}_h : R(x) \rightarrow R$ parameterized by $h \in R$. The action of ev_h is specified on representatives whose numerators and denominators are coprime—precisely the case where removable discontinuities have been algebraically eliminated—so evaluation reduces to syntactic substitution after canonical reduction (e.g., $\text{ev}_3(x)=3$), yielding a well-defined element of R and ensuring representation-independence.

Undergraduate level. We build $R(x)$ concretely as reduced pairs (p,q) with $p,q \in R[x]$ and $\gcd(p,q)=1$; ring operations incorporate Euclid’s algorithm so results



remain reduced. Because elements are *not* equivalence classes but normalized pairs, we define a single homomorphism ev directly on ring elements. Further we allow the shorthand $ev(x)=h$, where varying h implicitly varies the function definition ev . This mirrors elementary algebra while preserving the formal guarantees.

High-school level. We merely make an order of operations assertion that substitution has lower precedence than other arithmetic. In other words: **perform algebraic simplification before substitution**. In practice, this forces cancellation to occur prior to plugging in numbers, eliminating $0/0$ artifacts and yielding exception-free plotting. Despite its simplicity, this discipline aligns naturally with type-theoretic views of evaluation and with asymmetric parsing/rewriting relations that extend beyond mere equivalence.

Across all levels, formal graphs separate *indeterminates* from *values*, replace ad-hoc casework with homomorphisms, and offer a principled, calculus-free path to graphing expressions that would otherwise exhibit removable singularities. We provide worked examples (including difference quotients) and argue for the method's clarity, robustness, and pedagogical value.

Key Words: Graphing, Expressions, Formal Variables.

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Anti-Involutions of Dual Elliptical Quaternions

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ABSTRACT

Quaternions were defined by W.R. Hamilton in 1843. This facilitated the study of spherical rotations in three-dimensional real space. With the help of a small generalization of real quaternions, it is possible to study rotational motions on an ellipsoid rather than a sphere. To do this, a suitable quaternion can be defined for each ellipsoid by defining the inner product and vector product, in accordance with the metric that treats the given ellipsoid as a sphere. This quaternion is called an elliptical quaternion. Real quaternions are a special case of elliptical quaternions. Elliptical quaternions whose components are dual numbers are called dual elliptical quaternions. Dual quaternions are quaternions that have applications in mechanics, such as slip and screw operators.

In this study, some properties and anti-involutions of dual elliptical quaternions on the field of real numbers are given.

Key Words: Quaternion, elliptical quaternion, dual elliptical quaternion, involution, anti-involution.

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Involutions of Dual Elliptical Quaternions

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ABSTRACT

Quaternions were defined by W.R. Hamilton in 1843. This facilitated the study of spherical rotations in three-dimensional real space. With the help of a small generalization of real quaternions, it is possible to study rotational motions on an ellipsoid rather than a sphere. To do this, a suitable quaternion can be defined for each ellipsoid by defining the inner product and vector product, in accordance with the metric that treats the given ellipsoid as a sphere. This quaternion is called an elliptical quaternion. Real quaternions are a special case of elliptical quaternions. Elliptical quaternions whose components are dual numbers are called dual elliptical quaternions. Dual quaternions are quaternions that have applications in mechanics, such as slip and screw operators.

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Applications on (p, q) - analog of the generalized hyperharmonic polynomials of order v

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ABSTRACT

In this study, new identities and equalities have been established by employing the (p, q) -analog of generalized hyperharmonic polynomials of order v together with the

(p, q) -analogs of certain special numbers. Building on existing definitions and the

generating function, we have derived new summation formulas that demonstrate the interconnections between these families of polynomials and various sequences of special numbers. Several examples are provided to illustrate how the obtained equalities simplify under specific parameter choices and how they reduce to well-known classical results in particular cases.

The results obtained in this study are expected to have significance not only within the scope of pure mathematical research but also in related areas such as special functions, number theory, and algebraic combinatorics. Moreover, they are anticipated to contribute to a deeper understanding of how generalized hyperharmonic structures interact with other important mathematical objects and frameworks.

In addition, the new identities and formulas presented in this study provide further opportunities to analyze and interpret the algebraic and combinatorial properties of these polynomials in a broader context. Consequently, this research contributes to the advancement of combinatorial number theory through new equalities based on (p, q) -analogs, while at the same time opening new perspectives and potential

research directions in analytic combinatorics, number theory, and applied mathematics. It therefore serves as a step toward unifying diverse approaches and highlighting the applicability of (p, q) -analog methods to a wide spectrum of

mathematical problems.

Key Words: (p, q) -analog, the hyperharmonic number, generating function.



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ANALYSIS



Fefferman-Stein inequalities for the Riesz Bessel transform

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ABSTRACT

In this article, we establish the Fefferman-Stein inequalities for the B-maximal operator and Riesz Bessel transform associated with generalized translate operator generated by Laplace Bessel differential operator.

Key Words: Generalized translate operator, Maximal operator, Riesz Bessel transform.

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A Study on (p, q) -Bernstein-Chlodowsky-Durrmeyer Operators: One and Two Variables Approximation Properties

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ABSTRACT

This paper introduces a Chlodowsky-type variant of the (p, q) -Bernstein-Durrmeyer operators and explores their approximation properties in detail. The moments of the proposed operators are derived using the (p, q) -Beta and Gamma functions. The convergence behavior is established through the classical Korovkin-type approximation theorem. Additionally, the rate of convergence is examined with the help of the modulus of continuity and Peetre's K -functional. Numerical examples and graphical illustrations are also presented to demonstrate the effectiveness of the operators in approximating both univariate and bivariate continuous functions.

Key Words: (p, q) Bernstein Durrmeyer operator, Modulus of continuity, Peetre K functional.

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On the boundedness of the Multilinear Generalized Translate Operator

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ABSTRACT

In this paper, we will study the generalized shift operator corresponding to the solution of the Bessel differential equation. We will then define this operator as multilinear and show its boundedness in L_p spaces on the positive real line.

Key Words: Generalized shift operator, Bessel equation, Lebesgue spaces.

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Quantum Hardy-Littlewood Maximal Operator in the Quantum Lebesgue Spaces

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ABSTRACT

In this talk, we investigate the boundedness properties of the quantum Hardy-Littlewood maximal operator $q - M$ within the framework of quantum Lebesgue spaces $L_p(R, d_q)$.

Building upon recent advancements in q -calculus, this study aims to generalize and extend the Hardy-Littlewood maximal operator M concerning integral inequalities to the quantum setting. The research will focus on rigorously establishing conditions under which certain q -integral inequalities remain bounded in quantum Lebesgue spaces $L_p(R, d_q)$, utilizing techniques in the quantum calculus known as q -calculus.

Expected outcomes include novel boundedness of the quantum Hardy-Littlewood maximal operator $q - M$ and quantum analogues of well-known inequalities such as quantum Hölder inequality and quantum Minkowski inequality, contributing significantly to the field of quantum harmonic analysis and its potential applications in mathematical physics, especially quantum mechanics.

Keywords: q -integral, quantum Hardy-Littlewood maximal operator, quantum Lebesgue spaces.

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Quantum Hardy-Littlewood Maximal Operator and Quantum Riesz Potential Operator in the Quantum Morrey Spaces

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ABSTRACT

In this presentation, we focus on the boundedness properties of certain integral operators including the quantum Hardy-Littlewood maximal operator $q - M$ and quantum Riesz potential operator $q - I_\alpha$ within the framework of quantum Morrey spaces $\mathcal{L}_{p,\lambda}(R, d_q)$. Building on recent developments in q -calculus and quantum analysis, the study aims to extend classical results on quantum integral inequalities to the quantum Morrey setting.

The primary objective is to rigorously establish necessary and sufficient conditions for the boundedness of these operators on quantum Morrey spaces $\mathcal{L}_{p,\lambda}(R, d_q)$ using geometric characterization of balls in the quantum measure space (R, d_q) . Expected outcomes include the developments of new boundedness theorems and are anticipated to contribute significantly to the field of quantum harmonic analysis and its applications in mathematical physics, especially quantum mechanics.

Keywords: q -integral, quantum Hardy-Littlewood maximal operator, quantum Riesz potential operator, quantum Morrey spaces.

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TRAPEZOID-TYPE INEQUALITIES INVOLVING FRACTIONAL CONFORMABLE INTEGRALS VIA CO-ORDINATED S-CONVEX FUNCTIONS

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ABSTRACT

This study investigates generalized trapezoid-type inequalities via fractional conformable integrals in the context of co-ordinated s-convex functions. Using the notions of convexity and its extensions, we derive new inequalities that broaden the scope of classical Hermite–Hadamard and trapezoid-type results. The approach employs well-known inequalities such as Hölder and power-mean to establish generalized bounds for mappings whose second-order partial derivatives are s-convex in the co-ordinates.

The significance of these results lies in their generality: they not only unify previously known inequalities but also provide sharper estimates in fractional settings. Moreover, the established inequalities reduce to well-known cases in the literature, including those based on Riemann and Riemann–Liouville fractional integrals. The generalized conformable fractional calculus framework further strengthens the applicability of these results to a wide class of functions, while retaining consistency with classical calculus in the limiting cases.

Such results are potentially useful in various fields such as optimization, numerical analysis, applied sciences, and mathematical inequalities theory. The novelty of this work stems from extending trapezoid-type inequalities to the fractional conformable integral setting, thereby enhancing the theoretical framework of convexity-related inequalities.

Key Words: Trapezoid-type inequalities, s-convex functions, fractional conformable integrals, Hermite–Hadamard inequality, Riemann–Liouville fractional integrals.



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APPLIED MATHEMATICS



Solving Systems of Nonlinear Differential Equations: An Approximation Methodology

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ABSTRACT

In this study, we examine the Hermite collocation method, previously used in the study of [1] and [5], as a powerful tool for the numerical analysis of differential equation systems such as the Chen system. The Chen system, first investigated in [2], known for its ability to handle chaotic dynamic systems, utilizes a set of differential equations that provide a robust framework for modeling various real-world phenomena [4]. We emphasize the significance of the Hermite collocation method (HCM) in this context, particularly its ability to achieve high accuracy by incorporating both function values and their derivatives at specified collocation points. We also present a comparative analysis with other numerical methods [3],[6], highlighting the superiority of our approach in terms of computational ease and accuracy. Our findings suggest that pairing the Chen system with the HCM can be a viable alternative for researchers and practitioners dealing with chaotic systems governed by differential equations.

Key Words: Chen system, Hermite collocation method, Numerical analysis, Systems of nonlinear differential equations,

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Constructing Harmonic Green Functions in Polydomains

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ABSTRACT

In this work, we extend classical methods for constructing harmonic Green functions from planar domains to higher-dimensional polydomains. Specifically, we investigate three distinct approaches: the formulation via the Schwarz boundary value problem for holomorphic functions in several complex variables, the exploitation of conformal-type invariance properties adapted to polydomains, and a generalization of the Schwarz reflection principle suitable for domains whose boundaries consist of Cartesian products of circles and line segments. For each method, explicit constructions are provided for selected polydomains, illustrating the theoretical framework with concrete examples. This study offers a unified and constructive treatment of Green functions in polydomains, bridging techniques from classical complex analysis with higher-dimensional settings.

Key Words: Green functions, Schwarz reflection principle, polydomain.

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COMPARISON OF AN EPIDEMIOLOGICAL MODEL FROM A DETERMINISTIC AND STOCHASTIC PERSPECTIVE

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ABSTRACT

In this study, we propose an extended SIR-type epidemic model that integrates behavioral and environmental factors into the nonlinear incidence rate, aiming to provide a more realistic framework for analyzing infectious disease dynamics. Within the deterministic setting, we calculate the basic reproduction number (R_0) using the next-generation matrix method and investigate the local stability of both disease-free and endemic equilibria. These analytical results highlight how environmental and social feedback mechanisms alter epidemic thresholds and equilibrium behavior compared to the classical SIR formulation.

To account for the inherent randomness of epidemic processes, the model is further reformulated within a discrete-time Markov chain (DTMC) framework. Transition probabilities are explicitly derived, enabling a probabilistic interpretation of epidemic evolution. This stochastic perspective allows for the analysis of variability in outbreak size, extinction probabilities, and the timing of epidemic peaks. Numerical simulations are conducted to validate the theoretical findings and to compare the deterministic trajectories with multiple realizations of the stochastic process.

The results indicate that while deterministic models provide a baseline understanding of epidemic dynamics, stochastic models capture essential fluctuations, particularly in small populations or near the epidemic threshold. Such variability significantly influences the long-term outcomes of disease spread, demonstrating the necessity of incorporating stochasticity for accurate predictions.

Overall, the dual analysis presented here contributes both methodological and practical insights. By combining deterministic stability analysis with stochastic simulation, the study offers a more comprehensive perspective on epidemic modeling, enhancing its relevance for the design of public health strategies and intervention policies.



Key Words: Markov chains, SIR model, stochastic modelling.

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Solution of Differential equations whose coefficients are polynomial from first degree by using laplace transform

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ABSTRACT

In this study, the solution of linear ordinary differential equations with coefficients of first degree polynomials has been found using the Laplace transform and a formula was obtained for the most general case.

Key Words: Laplace transform, First degree polynomials, First order equation.

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The Bessel equation and its alternative analytic solution

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ABSTRACT

This paper discusses an alternative solution to the analytical solution of the Bessel equation, which is widely used in physical problems such as wave propagation and quantum mechanics, particularly in heat conduction problems. An alternative solution to the Bessel differential equation was obtained by applying the Fourier Transform method. This solution is based on the Dirac delta distribution, rather than the Bessel functions of the first and second kinds. We also used the series method to obtain the solution to the equation resulting from the Fourier transform. The resulting solution was substituted into the Bessel equation to demonstrate the validity and accuracy of the proposed method.

Key Words: Laplace transform, First degree polynomials, First order equation.

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MULTI-CRITERIA ERGONOMIC RISK ASSESSMENT OF CLEANING OPERATIONS USING PYTHAGOREAN FUZZY-VIKOR AND POSTURAL ANALYSIS METHODS

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ABSTRACT

The use of process-based, dynamic risk assessment approaches is crucial for occupational health and safety. Cleaning tasks are often characterized by repetitive movements, awkward postures, prolonged static positions, and forceful exertions, all of which contribute significantly to the development of work-related musculoskeletal disorders (WMSDs). The prevention of musculoskeletal disorders (MSDs) among cleaning staff requires comprehensive ergonomic assessments and decision-making models that reflect real-world complexities. This study proposes an integrated risk assessment methodology combining Rapid Entire Body Assessment (REBA), Rapid Upper Limb Assessment (RULA), and the Pythagorean Fuzzy VIKOR (PF-VIKOR) approach to prioritize ergonomic hazards in cleaning operations. A total of eighteen distinct cleaning tasks, identified through direct field observations in a high-traffic airport service building, were analyzed using REBA and RULA to determine their associated ergonomic risk levels. The findings revealed that nearly all evaluated tasks posed moderate to very high risks. To enhance the objectivity and granularity of risk prioritization, a multi-criteria decision-making framework using PF-VIKOR was applied, incorporating expert input from three decision-makers and utilizing criteria such as severity, occurrence, and activity intensity. The fuzzy logic-based model allowed for managing the inherent uncertainties in ergonomic evaluations and provided a more nuanced ranking of



hazardous tasks. The proposed methodology successfully ranked 18 hazardous working positions, with pushing of suspended scaffolding carrier legs (R18, $Q=0.000$) identified as the highest priority risk, followed by cleaning of toilet ceilings (R6, $Q=0.216$) and cleaning of office floors (R10, $Q=0.278$). The hybrid approach demonstrated enhanced capability in handling subjective assessments and providing robust risk prioritization compared to traditional methods. The results demonstrate that tasks such as cleaning ceilings, pushing scaffolding, and window cleaning exhibit the highest ergonomic risk scores. The proposed hybrid approach not only improves the precision of ergonomic risk assessments but also supports evidence-based decision-making in the design of preventive interventions. Ultimately, this methodology facilitates the development of safer work environments, contributing to the reduction of injury-related absenteeism and improving overall worker health and operational efficiency.

Key Words: Ergonomics, Multi-Criteria Decision Making, Pythagorean Fuzzy

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WHY IS FUZZY SIMILARITY

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ABSTRACT

This talk will explain the historical development of the concept of similarity and its transition to the concept of degree of similarity, with justification. We will then calculate the degree of similarity between two different groups using the degree of similarity method. Furthermore, we will reveal the degree of similarity between elements within a group. After presenting this information, we will present the results of a TÜBİTAK project using the degree of similarity method we developed..

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Direct and Inverse Problems for the System of Electromagnetoelasticity Differential Equations in the Linear Approximation

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ABSTRACT

This study presents a rigorous mathematical formulation of both the direct and inverse problems for the system of differential equations of electromagnetoelasticity (EME) in the linear approximation. This system describes the propagation of coupled electromagnetic and elastic waves in piezoelectric crystalline media. The original system, consisting of nine equations, is transformed into a t-hyperbolic symmetric Friedrichs system of the first-order form comprising fifteen equations. Such a transformation provides a symmetric structure, which significantly facilitates subsequent analytical investigations.

Within this framework, the initial–boundary value Cauchy problem for the direct case is analyzed. An appropriate class of Hilbert- and Sobolev-type functional spaces is identified, in which the well-posedness of the direct problem is rigorously established. Theorems on existence, uniqueness, and continuous dependence of the solution on the initial data are proven.

In the same functional setting, the linearized inverse problem is studied, aimed at reconstructing the elastic and piezoelectric coefficients of an anisotropic medium with cubic structure from data on external excitations and system responses. An explicit transformation matrix is constructed that reduces the matrices corresponding to derivatives with respect to t and x_3 to diagonal form. Furthermore, theorems on the existence, uniqueness, and stability of solutions to the multidimensional inverse problem are formulated and established.

A constructive method is developed for solving both the direct and inverse problems in the linearized formulation, which highlights the novelty and significance



of the proposed approach. The obtained results contribute to the theoretical understanding of wave propagation in coupled systems and provide methodological tools for the analysis of inverse problems.

Key Words: Electromagnetoelasticity, t-hyperbolic symmetric Friedrichs system of the first order, direct problem, inverse problem, initial boundary value problem, elastic and piezoelectric moduli, piezo crystalline media.

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A Non-Uniform Rational B-spline Approach for Nonparametric Cumulative Distribution Function Estimation

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ABSTRACT

This study introduces a new nonparametric method for estimating cumulative distribution functions by employing the Non-Uniform Rational B-spline (NURBS) framework. In the proposed approach, the control points are derived from the empirical distribution function, while the knot structure is determined through an information-based criterion, and the weights are optimized under monotonicity constraints. This design ensures that the resulting estimator preserves the fundamental properties of distribution functions and provides a smooth, flexible approximation. The theoretical analysis establishes the uniform convergence of the estimator, and its finite-sample performance is examined through extensive Monte Carlo simulations. The findings demonstrate that the NURBS-based estimator achieves consistently lower approximation errors compared with the classical Bernstein polynomial estimator. In addition, the NURBS estimator shows a remarkable advantage when handling complex, skewed, or multimodal distributions. Applications to real datasets further highlight the adaptability and flexibility of the method, confirming its usefulness in modern statistical modelling and practical data analysis. An important advantage of the proposed approach is its ability to deliver accurate results even with low polynomial degrees, reducing the risk of overfitting and computational burden. Overall, the NURBS framework provides a powerful and versatile alternative for nonparametric distribution function estimation, combining theoretical rigor with practical efficiency.

Key Words: NURBS, cumulative distribution function, Monte Carlo simulation.



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A Cubic Spline Approach for a Crack Problem in a Transversely Isotropic Elastic Layer

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ABSTRACT

This study addresses the numerical solution of singular integral equations using cubic spline interpolation. The method has advantages in accuracy and stability since cubic splines divide the interval into smaller subintervals and construct piecewise cubic polynomials that ensure continuity of the function, as well as its first and second derivatives, across the knots. Cubic spline methods can be successfully applied to the solution of the singular integral equations that frequently arise in fracture mechanics.

In this study, an application of the cubic spline interpolation to the singular integral equation leading from a crack problem in a transversely isotropic elastic layer is presented. The crack problem is formulated within the framework of elasticity theory and the geometry of the problem. The singular integral equation with the Cauchy kernel is obtained. The cubic spline approximation is applied to the singular integral equation to obtain the stress intensity factors at the crack edges. The effectiveness of the cubic spline method is examined by comparing the obtained results with those derived from Lobatto-Chebyshev and Gauss-Chebyshev quadrature methods reported in the literature. The comparison indicates that spline-based methods can serve as a reliable and efficient tool for the numerical solution of singular integral equations in elasticity and fracture mechanics.

Key Words: Crack, cubic spline, singular integral equation.

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Integrating Noise Analysis and Fuzzy Decision Trees for Geomtry-Aware Optimization of Solid-State Nanopores

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ABSTRACT

Solid-state nanopores are widely used in single-particle and biomolecule detection due to their high sensitivity and label-free operation. However, the choice of pore geometry critically influences signal magnitude, waveform, and robustness against variations in particle charge, size, and trajectory. While prior studies have highlighted the role of pore shape in resistive-pulse sensing [Yılmaz et al., ChemistrySelect 2021], and fuzzy decision frameworks have been applied to optimize sensor networks under uncertainty [Lizarraga et al., ACM T-EC Systems 2019], there remains a gap in unifying performance metrics with decision-making tools for nanopore design.

In this study, we employ finite-element simulations to compare three main geometries under standardized electrochemical conditions. Beyond current modulation ($\Delta I/I_0$), we systematically evaluate low-frequency $1/f$ noise and signal-to-noise ratio (SNR) as geometry-dependent parameters. To rationalize geometry selection, we introduce a fuzzy decision tree (FDT) that integrates charge sensitivity, size sensitivity, trajectory impact, and noise robustness into a single interpretable framework. The results show that hourglass-shaped pores consistently provide superior size discrimination, cigar pores excel in charge sensitivity albeit with trajectory susceptibility, and conical pores offer balanced charge-focused detection but suffer from higher low-frequency noise.

By combining detailed computational modeling with noise analysis and fuzzy inference, our approach provides a structured, data-driven methodology for nanopore geometry optimization. This framework supports the rational design of nanopore-based biosensors and contributes to bridging the gap between physical signal modeling and adaptive decision strategies.



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Key Words: Nanopore sensing, resistive pulse, pore geometry, fuzzy decision tree, size discrimination, charge sensitivity.

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Schur Stability Analysis of Families of Polynomials: Legendre Polynomial

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ABSTRACT

In this study, we investigate the Schur stability of the matrix families obtained from the Legendre polynomials.

The Legendre polynomials $P_n(x)$ are defined on the interval $[-1,1]$ and satisfy the recurrence relation $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$, $n \geq 1$ with $P_0(x) = 1$ and $P_1(x) = x$ [1]. Due to their definition over the interval $[-1,1]$, Legendre polynomials satisfy the conditions of Schur stability. On the other hand, there are methods and algorithms that extend the Schur stability interval of the Schur stable matrices [2,3]. These methods and algorithms based on the continuity theorems and the parameter ω which indicates the quality of Schur stability of the matrices, is defined as $\omega(A) = \|H\| \geq 1$, where $H = H^* > 0$ is satisfy the Lyapunov matrix equation $A^*HA - H + I = 0$ [4-6].

In this study, matrix families $\mathcal{L}_N$ were introduced with linear combinations of the central matrices A which is generated from the Legendre polynomials and the perturbation matrices B which is generated from $E_{Ni}, i \leq N$, respectively. With methods and algorithms, the intervals were obtained. These intervals guarantee the Schur stability of the matrix families centered on the Legendre polynomial. Some properties and results about these matrix families were introduced. Finally, examples related to the these matrix families were given.

Key Words: Schur stability, continuity theorems, Legendre polynomials.

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PRIORITIZING PROCESS SAFETY MANAGEMENT ELEMENTS IN OFFSHORE OPERATIONS: AN INTEGRATED AHP–TOPSIS APPROACH

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ABSTRACT

Process Safety Management (PSM) systems represent a cornerstone in high-hazard industries for systematically identifying, evaluating, and controlling operational risks. Their importance is particularly pronounced in offshore industrial environments, where complex technological processes, harsh environmental conditions, and elevated safety criticality amplify the consequences of potential failures. The successful implementation of PSM requires not only comprehensive coverage of safety elements but also a clear prioritization strategy that reflects organizational objectives, resource limitations, and the unique characteristics of offshore operations.

This study develops a prioritization framework for PSM elements by integrating two multi-criteria decision-making (MCDM) techniques: the Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). AHP is employed to determine the relative importance of evaluation criteria, while TOPSIS ranks alternative PSM elements according to their proximity to the ideal solution. This combined approach addresses the subjectivity and complexity inherent in safety-related decision-making, ensuring a more transparent, systematic, and mathematically robust process. The proposed framework supports organizations in designing phased and context-sensitive implementation strategies that align safety investments with operational goals and regulatory standards. By optimizing resource allocation and clarifying decision priorities, the methodology enhances both the effectiveness and efficiency of PSM deployment. The



contribution is especially relevant for offshore industries, where constrained resources, dynamic conditions, and the high cost of safety failures demand structured decision-making tools.

Ultimately, this research advances the integration of decision science into safety management and offers a practical pathway toward improving overall safety performance.

Key Words: AHP, Process Safety Management, TOPSIS

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Schur Stability Analysis of a Two-Economy Macroeconomic Model

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ABSTRACT

Real-world problems are usually modeled by difference or differential equations under certain constraints. In many situations, it is important to understand the behavior of solutions without solving the model explicitly. The stability analysis of equilibrium points is one of the main tools for this purpose. A common approach is based on spectral criteria. However, it is known that the study of spectral properties for systems with non-symmetric coefficients is an ill-posed problem [1,2].

Another approach is related to the Lyapunov Matrix Equation (LME) and the existence of a positive definite solution. Still, the existence of such a solution does not directly give information about how strong the stability is. For this reason, a stability parameter defined through the solution of the LME has been considered. This parameter provides information both on the stability itself and on the quality of stability. Moreover, by using continuity theorems, the sensitivity of this parameter can be studied, which makes it possible to measure how resistant the system is against external effects [1–4]. In this way, stability intervals can be obtained for matrix families, and these intervals can sometimes be extended with the help of certain methods and algorithms [5,6].

In this study, we focus on a two-economy macroeconomic difference model [7,8]. The stability and sensitivity of the model are examined by using the Schur stability parameter. This analysis helps to understand the interaction between the two economies, their equilibrium states, and their reactions to shocks in a mathematical way. The conditions under which the model is stable on parametric input elements, in a sense as an inverse problem, are investigated and stability



intervals are determined. Calculations for different parameter values were carried out in Python, and the results were supported with graphical illustrations.

Key Words: Schur stability, continuity theorems, stability parameter, macroeconomic model.

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Decision Making About Installation Locations of Wind Power Plants in Çanakkale by Using Hierarchical Clustering for Spatial Point Data

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ABSTRACT

Spatial clustering is a valuable method in computational geometry that involves grouping spatial point data into clusters based on their proximity to each other. It helps to identify patterns which can be used in various fields such as Geographic Information Systems, healthcare, ecology, urban planning and energy. Wind energy is one of the most important renewable energy sources and is used to increase energy efficiency in our country. In order to increase the efficiency of wind energy, the installation locations of wind power plants (WPPs) must be determined optimally. In this study, Çanakkale province was taken into consideration since it is the most windy settlement in our country. Hierarchical Clustering method was applied to spatial point data in order to determine the optimal installation locations of the WPPs in Çanakkale. The Çanakkale province was divided into 4 km² regions and 157 suitable grids were determined using ArcGIS, taking into account official and literature constraints. In the study, a dataset consisting of 157 grids displayed over Çanakkale province, based on 7 different criteria, was prepared for clustering analysis through data preprocessing. Ward's hierarchical clustering algorithm was implemented with R package ClustGeo (version 2.0) developed by Chavent et al. [1]. It was demonstrated that Ward's hierarchical clustering algorithm can be used to determine optimal WPPs installation locations in Çanakkale province by identifying optimal regions. Then, a priority ranking of the optimal WPPs installation locations was obtained by applying Multi-Criteria Decision Making (MCDM) methods to the spatial point data within these regions.

Key Words: Spatial point data, Ward's hierarchical clustering algorithm, Multi-Criteria Decision Making (MCDM) methods, renewable energy.



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Well-Posedness of Nonlinear Stochastic Wave Propagation Model with Additive Noise

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ABSTRACT

This study provides a comprehensive investigation on the unique solvability and stability of the model for a class of stochastic nonlinear wave equations incorporating additive noise term, along with the numerical applications to support theoretical findings. We apply analogous motivation of well-established theoretical framework of deterministic nonlinear wave system in addition with suitable usage of Ito calculus to handle the complexities caused by random fluctuations. Main theoretical statements utilizes the Banach Fixed Point Theorem, adapted to the stochastic setting, to demonstrate the unique solvability result of these systems. This procedure demonstrates strong connection with classical methods while it effectively handles stochastic perturbations. In addition to theoretical investigation, the present study shows the practical aspects of the utilised method by numerical applications. We employ the Du Fort-Frankel finite difference scheme, to discretize the stochastic nonlinear wave model. Numerical results are presented to illustrate the dynamic behavior of the nonlinear stochastic wave solutions under the effect of additive noise. These theoretical and numerical findings contribute both structural and applicational aspect of nonlinear stochastic wave propagation model and the present work exhibits practical implications and applicability of the proposed model in simulation of real-world phenomena in which random effects are essential.

Key Words: Stochastic nonlinear wave equation, additive noise, fixed point thoery.

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On The Mersenne Polynomials

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ABSTRACT

In this article, first, a hypergeometric equation of the Mersenne polynomials is constructed. Then, the hypergeometric form of Mersenne polynomials is obtained by solving this hypergeometric equation. Subsequently, the properties of Mersenne polynomials will be examined by presenting their recurrence relation, integral representation, and various types of explicit formulas and generating function relations.

Key Words: Mersenne polynomials, Hypergeometric equation, Hypergeometric function.

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VARIATION OF PARAMETERS AND HÖLDER STABILITY OF MATRIX DIFFERENTIAL SYSTEMS WITH INITIAL TIME DIFFERENCE

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ABSTRACT

The method of variation of parameters formulae (VPF) has been beneficial techniques in the qualitative theory of nonlinear matrix differential equations since it is a practical tool in the investigation of the properties of solutions. Recently, the study of nonlinear matrix system of equations and initial value problems without an initial time difference (ITD) has been initiated and the corresponding theory of differential inequalities have been investigated. Below, we shall derive VPF showing the relationship between: unperturbed matrix differential systems with different initial conditions and unperturbed and perturbed matrix systems with different initial conditions.

Key Words: Differential equations, initial time difference Hölder stability, variation of parameters.

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VARIATION OF PARAMETERS AND LIPSCHITZ-TYPE STABILITY IN UNPERTURBED DIFFERENTIAL SYSTEMS WITH INITIAL TIME DIFFERENCE

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ABSTRACT

In this study, we explore the connection between unperturbed differential systems that exhibit variations in both their initial conditions and the starting time. Through the application of variation of parameters methods, we construct integral representations that reveal this relationship. To formulate Lipschitz-type stability criteria under the presence of an initial time shift in nonlinear differential systems, it is essential to employ the variational framework derived from the unperturbed model.

Key Words: Differential equations, initial time difference Lipschitz stability, variation of parameters.

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A fixed point approach to the Dirichlet problem for the generalized Beltrami equation

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ABSTRACT

In this talk, we discuss the solvability of the Dirichlet problem for the generalized Beltrami equation in the unit disc. By employing properties of some integral operators and applying the Banach fixed point theorem, we demonstrate the existence and uniqueness of the solution.

The analysis begins with a review of the Dirichlet problem for the inhomogeneous Cauchy–Riemann equation. Next we recall some properties of the singular integral operators, which play a central role in reformulating the problem into an equivalent integral equation. Finally we investigate the Dirichlet problem for the generalized Beltrami equation in the unit disc. This problem is transformed into an equivalent integral equation using the Pompeiu operators. The resulting formulation leads to a system of integro-differential equations in a Hölder space. By defining a suitable operator and establishing norm estimates for the integral operators, it is shown that the mapping is contractive under the constraint. Applying the Banach fixed point theorem, this guarantees the existence and uniqueness of the solution.

The method can be adapted to study the solvability of the generalized Beltrami equation under alternative boundary conditions. Since explicit solutions of the inhomogeneous Cauchy–Riemann equation are available in various settings, the approach can be extended to a broader class of boundary value problems.

Key Words: Dirichlet problem, Generalized Beltrami equation, Banach fixed point theorem.

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COMPUTER SCIENCES



LEVERAGING GENERATED SYNTHETIC DATA TO PRESERVE DATA PROPERTIES AND VALIDATE CLUSTERS USING CLUSTER NUMBER OPTIMIZATION TECHNIQUE

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ABSTRACT

In the realm of unsupervised machine learning, clustering algorithms are essential for uncovering hidden patterns and structures within datasets. However, the effectiveness of these algorithms is often hindered by the availability of substantial and representative data. Small datasets, a common challenge in many fields, can significantly compromise the performance of clustering models due to increased susceptibility to overfitting and limited generalizability. To address these challenges, this paper explores the utility of synthetic data generation techniques as a strategic approach to augment small datasets, thereby enhancing the robustness and accuracy of clustering algorithms. We specifically focus on the application of synthetic data generation to identify optimal cluster numbers using popular unsupervised clustering algorithms. Through comprehensive experiments and analysis, we demonstrate that synthetic data, carefully generated to preserve the statistical properties of the original small datasets, can create a more enriched data environment. This enriched environment facilitates more effective training, validation, and testing of clustering models. We discuss methods for generating synthetic data that closely mimic real-world distributions, enabling more accurate model tuning and hypothesis testing in unsupervised learning scenarios. Our findings underscore the potential of synthetic data to overcome the limitations of small datasets and advance the field of machine learning by providing a more versatile and resilient analytical framework for clustering tasks. This study aims to guide researchers and practitioners in leveraging synthetic data generation to enhance the analytic



capabilities of unsupervised clustering algorithms, ultimately leading to more reliable outcomes across various applications.

Key Words: Synthetic data generation, Elbow method for clustering, DBSCAN clustering.

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A NEW ALPHA POWER RAYLEIGH WEIBULL DISTRIBUTION WITH ITS APPLICATIONS TO SIMULATED AND REAL DATA

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ABSTRACT

In this article, a statistically-enhanced lifetime density, with the name of the Alpha Power Rayleigh-Weibull Distribution (APRWD) is proposed. Emerging processes have been applied to the formulation of this distribution from a systematic generalization of the Rayleigh and Weibull families by employing the Alpha Power transformation technique. The parameter (α) as a tunable shape is introduced by this extension that enhances the distribution's responsiveness to diverse data structures and failure mechanisms. The inclusion of the alpha parameter grants the model flexibility to characterize a wide spectrum of hazard behaviors. Such as, monotonic increases or decreases, and configurations that are more complex, like the bathtub shape. The probability density function, the cumulative distribution function, and the survival function are the three main statistical functions that are included in the analytical development of the APRWD. We use both the Maximum Likelihood Estimation (MLE) and Ordinary Least Squares (OLS) methods to provide accurate parameter estimations. Additionally, meticulously planned simulation studies are used to test the estimators' accuracy and resilience. We apply the APRWD to actual datasets in order to validate the model's practicality. APRWD frequently produces a better fit and shows more modeling precision when handling lifespan and reliability data, according to comparisons with traditional models.



Key Words: moments about the origin, survival function, Rayleigh Weibull distribution, alpha-power family, and Monte Carlo simulation.

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Automatic Lesion Segmentation in Dental Bitewing Radiographs: Comparative Analysis of U-Net, Attention U-Net, Mask R-CNN, and YOLOv11n-seg Models

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ABSTRACT

Bitewing radiographs provide high diagnostic value in the early diagnosis of dental diseases, particularly in the detection of caries and restorative materials (Lee et al., 2021). In this study, the performance of four different deep learning-based architectures was compared for automatic lesion segmentation in dental bitewing radiographs. The models used were U-Net, Attention U-Net, Mask R-CNN, and YOLOv11n-seg. The Dental Bitewing X-ray Dataset, which is openly available on Kaggle, was used, and color-coded mask annotations were converted to class ID format to make them suitable for model training. U-Net and Attention U-Net are pure segmentation models that perform pixel-level classification on the entire image, and the Dice coefficient and Intersection over Union (IoU) metrics were used to measure segmentation success. Similar approaches in the literature report high Dice scores in tooth segmentation (Dhar & Deb, 2022). Mask R-CNN and YOLOv11n-seg are object-based segmentation models that generate the location and pixel mask of each object and were therefore tested using COCO mAP@0.5:0.95 criteria.

The results show that the U-Net model achieved the highest pixel-based segmentation success with an average Dice = 0.8940 and IoU = 0.8104, while Attention U-Net ranked second with Dice = 0.8726 and IoU = 0.7766. Mask R-CNN achieved a mask mAP@0.5:0.95 of 0.6656 for mask segmentation, while the YOLOv11n-seg model achieved a box mAP@0.5:0.95 of 0.5313 and a mask mAP@0.5:0.95 of 0.4647, demonstrating high performance particularly in the filling-metal and dentin classes. However, it is noteworthy that both models showed low



success in the cavity class. This situation may be due to factors such as the loss of small lesion areas at low resolution, class imbalance, and the uncertainty of annotations. Furthermore, this low success rate in cavity detection from a clinical perspective may lead to missed opportunities for early diagnosis and delays in treatment planning. The findings show that U-Net-based architectures offer strong performance in pixel-level segmentation, while the YOLOv11n-seg model provides advantages in applications requiring real-time and object-based segmentation. In the literature, approaches similar to YOLOv11n-seg, such as YOLO-DentSeg, provide high mAP and FPS values in real-time dental diagnosis systems (Hua et al., 2025).

Key Words: Dental bitewing radiographs, Deep learning segmentation, Caries detection

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AI-Enhanced Intrusion Detection Using the T-Pot Honeypot System

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ABSTRACT

Honeypot-based systems provide a valuable means of monitoring attacker behavior and generating high-quality threat intelligence. This study introduces an artificial intelligence-supported intrusion detection framework built on the T-Pot honeypot platform, which consolidates multiple honeypot services into a unified monitoring environment. The experimental setup subjected the system to Distributed Denial-of-Service (DDoS), brute-force, and port scanning attacks, generating over 250,000 packets of traffic. Network traces were captured using Wireshark and Tshark, feature-engineered, and transformed into structured datasets for analysis. Several machine learning models were evaluated for attack classification. The Neural Network model achieved near-perfect performance with an accuracy of 99.9%, AUC of 1.000, precision/recall/F1 of 0.999, and MCC of 0.999. The SVM model also performed strongly with 97.3% accuracy and MCC of 0.959, whereas Naive Bayes yielded weaker results with 64.3% accuracy. These findings demonstrate that deep learning-based approaches can effectively distinguish diverse attack types, significantly outperforming classical models. Overall, the integration of artificial intelligence with the T-Pot honeypot system offers a robust methodology for proactive cybersecurity defense. The results highlight that neural networks, in particular, can deliver highly reliable intrusion detection in real-time environments, providing a scalable solution for protecting modern network infrastructures against evolving cyber threats.

Key Words: T-Pot, IDS, Cyberattack Classification, Artificial Intelligence.



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Passive Acoustic Localization of Surface Vessels Using Hydrophone Array Measurements

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ABSTRACT

Passive acoustic sensing is a critical approach for monitoring maritime traffic and estimating the relative position of surface vessels without relying on active sonar. In this study, we investigate distance categorization of ships using hydrophone recordings from the QiandaoEar22 dataset, a publicly available underwater acoustic database. The dataset provides ship-radiated noise signals labeled with three relative distance categories: Near (N), Medium (M), and Far (F). To exploit this information, we compute Power Spectral Density (PSD) features from the recorded signals and analyze the spectral energy distribution across different frequency bands. The extracted PSD features are then used to train machine learning models for classifying vessel distances into near, medium, and far categories. Our experimental results show that spectral energy patterns provide distinctive cues related to propagation loss and absorption effects, enabling reliable relative distance estimation. The proposed method demonstrates that PSD-based passive acoustic analysis can effectively distinguish between distance categories of maritime targets, offering a lightweight and interpretable solution for underwater acoustic localization. These findings suggest that PSD-derived features, coupled with supervised learning, are a promising tool for enhancing passive localization systems and maritime situational awareness in both civil and defense applications.

Key Words: statistical signal processing, underwater acoustic localization, machine learning



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A Mathematical Comparison of SLAM Families: Observability, Information-Theoretic Bounds, and Optimization Landscapes

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ABSTRACT

Simultaneous Localization and Mapping (SLAM) is a fundamental capability in autonomous robotics, enabling agents to estimate their trajectories while building maps of unknown environments. Several algorithmic families have been developed, including filter-based methods, particle-based approaches, optimization-based formulations, and density-based techniques.

Most comparative studies rely on trajectory-specific metrics such as Absolute Pose Error (APE) and Relative Pose Error (RPE). While useful, these do not fully explain the theoretical factors shaping performance. This study introduces a unified mathematical framework for comparison. First, observability and consistency are examined, showing how gauge freedoms cause inconsistency and how invariant formulations preserve stability. Second, information-theoretic bounds are explored through Fisher Information Matrices and Posterior Cramér–Rao Lower Bounds, revealing dataset-independent accuracy limits and the impact of graph connectivity. Third, the optimization landscape is analyzed, demonstrating how sparse solvers achieve scalability, while semidefinite relaxations provide certifiable global optimality under realistic noise conditions.

The analysis highlights trade-offs across SLAM families: particle-based methods remain effective in sparse measurement regimes, invariant filtering ensures robustness on Lie-group representations, and optimization-based approaches excel when loop closures provide strong conditioning. Density-based methods, while robust in repetitive environments, are sensitive to parameterization.



By bridging empirical evaluation with rigorous mathematical analysis, this framework provides principled guidelines for algorithm selection and the design of active SLAM strategies.

Key Words: SLAM, Observability, Fisher Information, Cramér–Rao Bound, Pose Graph Optimization, Autonomous Robotics.

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A Chaotic Image Encryption Algorithm Based on Knight Tour and White Shark Optimizer

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ABSTRACT

Because of the rise in data traffic and cyberthreats, it is now imperative to securely transmit and store digital images. High pixel correlation and large image sizes might be problematic for traditional approaches. Given their highly effective confusion and diffusion behaviour, Combining chaotic dynamics with meta-heuristic optimization, these hybrid methods exceed traditional limits and reduce pixel correlation. In this paper, a novel encryption technique is offered for color images that combines Lorenz Chaotic System, White Shark Optimizer, Chess Knight Tour and The Fisher-Yates shuffling algorithm.

In the confusion stage, image pixels are permuted using four separate Knight's Tour permutation cycles, each tied to a unique key fragment. This effectively reduces the correlation between pixel positions while maximizing the algorithm's key sensitivity. The Fisher-Yates shuffling algorithm is then applied to further reduce the correlation. The diffusion stage utilizes the Lorenz attractor, a chaotic system, to enhance the security of this stage, optimal values for the Lorenz system parameters (σ , ρ , β) are optimized using the White Shark Optimizer (WSO) algorithm. These optimized parameters obtained from the WSO will be used for final encryption using a chained XOR operation, ensuring that even a small key or pixel change affects the entire encrypted image.

Comprehensive experimental results and security analyses demonstrate that the proposed method is robust and effective against various cyberattacks. Even a pixel change to the plain image affects the entire encrypted output because of this complex approach, which essentially produces an avalanche effect. In particular,



entropy, correlation coefficient, and NPCR/UACI tests demonstrate strong resistance to traditional threats.

Key Words: Image Encryption, White Shark Optimization, Knight Tour, Fisher-Yates Shuffling, Chaotic Maps.

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Security Evaluation of IoT Systems in Smart Factories: A Fuzzy Data Envelopment Analysis Approach to RPL-Based Cyberattacks

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ABSTRACT

The digital transformation of Industry 4.0 has positioned smart factories as core drivers of efficiency, automation, and productivity. At the center lies the Internet of Things (IoT), enabling real-time monitoring, predictive maintenance, and intelligent decision-making, while also exposing critical infrastructures to advanced cyber threats. This study examines IoT vulnerabilities in smart factories, with a focus on attacks against the IPv6 Routing Protocol for Low-Power and Lossy Networks (RPL). Attacks such as DIS Flooding, Version Number Modification, Black Hole, and Sink Hole severely affect network stability and data integrity. Simulations using Contiki 3.0 and Cooja replicated real-world attack scenarios, with packet-level data analyzed via Foren6. The novelty of this research lies in applying Fuzzy Data Envelopment Analysis (Fuzzy DEA) to assess the efficiency and impact of cyberattacks. Unlike deterministic DEA, the fuzzy model accounts for uncertainty in IoT environments, offering a more realistic evaluation. Triangular fuzzy numbers derived from input-output variables enabled efficiency scoring across multiple α -cuts, allowing comparative analysis under different uncertainty levels. Results showed notable variations, revealing differential impacts of RPL-based attacks. The findings highlight the importance of uncertainty-aware approaches in IoT security analysis. Integrating Fuzzy DEA provides a systematic approach for prioritizing defense strategies, optimizing resource utilization, and facilitating proactive risk management. Practically, the results provide insights for strengthening smart factory resilience by identifying the most disruptive attack vectors. This work bridges the gap between IoT cybersecurity and performance evaluation, demonstrating that Fuzzy DEA is a valuable tool for analyzing cyber threats and informing future industrial protection strategies.

Key Words: IoT Security, Smart Factories, RPL Protocol, Cyberattack Simulation, Fuzzy Data Envelopment Analysis, Industry 4.0



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Customer Segmentation via Unsupervised Learning: Applications of K-Means and Hierarchical Clustering

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ABSTRACT

Customer segmentation constitutes a central application of applied mathematics within modern business analytics, providing a rigorous framework for classifying consumers according to demographic and behavioral attributes. This process not only enhances marketing efficiency, customer loyalty, and long-term profitability, but also underscores the mathematical foundations of data-driven decision-making systems. Formally, segmentation seeks to partition a population into distinct subgroups so as to minimize intra-group variance while maximizing inter-group heterogeneity, thereby improving predictive accuracy and strategic decision processes. Initially introduced in the early twentieth century as a simple demographic ranking method, segmentation has evolved considerably through the integration of statistical learning and machine-driven analytics. In contrast to traditional profiling approaches, clustering algorithms such as K-Means and Hierarchical Clustering allow for mathematically precise, dynamic, and scalable segmentation structures. The present study investigates the empirical implementation of these clustering techniques in the retail and e-commerce sectors, where personalization, optimized resource allocation, and robust decision-making are of critical importance. Data preprocessing was carried out through Microsoft SQL Server, followed by computational analysis and visualization in RStudio. The findings demonstrate that mathematically grounded segmentation models not only advance predictive accuracy in customer lifetime value estimation but also reinforce customer acquisition and retention strategies, cross-selling optimization, and targeted communication. Moreover, the results highlight the contribution of clustering methodologies to sustainable competitive advantage by fostering deeper customer engagement, enhanced profitability, and stronger brand loyalty.



Key Words: Customer segmentation, unsupervised learning, clustering algorithms, K-Means algorithm, SQL-Based preprocessing, R-Studio visualization.

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A Comparative Study of DEMATEL and AHP–TOPSIS in Maritime Accident Risk Factors

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ABSTRACT

Maritime accidents are among the most critical challenges in ensuring navigational safety, environmental protection, and efficient shipping operations. Identifying the main and sub-factors that contribute to these accidents is essential for developing effective prevention strategies. This study conducts a comparative analysis of the causal and influential relationships among accident-related factors using the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method [2] and evaluates their relative importance through the Analytical Hierarchy Process (AHP) [3] and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) approach [4]. The analysis is based on expert evaluations and prior research on ship accident causation [1]. A total of five main criteria and seventeen sub-criteria were examined. The DEMATEL analysis revealed that Structural Risk and Navigation-Related Factors belong to the cause group, while External Factors, Education and Experience, and Psychosocial Risks fall into the effect group. Among the sub-criteria, Technical Failures, Inadequate Maintenance, and Insufficient Supervision were identified as the most influential causal factors, whereas Traffic Density, Adverse Meteorological Conditions, and Deficiencies in Voyage Planning appeared as the least influential effect factors. In contrast, the AHP-TOPSIS analysis highlighted Education and Experience as the most critical main criterion [6]. Despite methodological differences, both approaches consistently emphasized Technical Failures as the top-ranking sub-criterion, underscoring the pivotal role of technical reliability in maritime safety [7]. The comparison indicates that DEMATEL effectively captures interdependencies and systemic causality among criteria [5], whereas AHP-TOPSIS focuses on priority rankings based on expert judgments of importance. The findings suggest that a combined methodological perspective provides deeper insights into both the direct and indirect impacts of accident-related risks. Ultimately, improving technical reliability, ensuring continuous training, strengthening inspection mechanisms, and mitigating external risk factors emerge as priority strategies for enhancing maritime safety. This comparative analysis not only enriches the academic literature on multi-criteria decision-making in maritime safety but also offers practical guidance for policymakers and practitioners aiming to reduce accident risks.

Key Words: Maritime accidents, DEMATEL, AHP-TOPSIS, risk analysis, multi-criteria decision-making



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AI-Driven Detection and Mitigation of Cyberattacks on Siemens S7-1200 PLC Devices in Industrial Control Systems

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ABSTRACT

The increasing reliance on Industry 4.0 technologies has rendered Industrial Control Systems (ICS) highly vulnerable to sophisticated cyberattacks. Programmable Logic Controllers (PLCs), as core components of ICS, are particularly exposed due to their network connectivity and real-time operational roles. This study focuses on the Siemens S7-1200 PLC device, where different attack scenarios—Distributed Denial of Service (DDoS), Man-in-the-Middle (MitM), and Start/Stop attacks—were simulated in a controlled environment and analyzed using machine learning techniques. During the DDoS simulations, the system encountered an overwhelming traffic load of more than 15,000 packets per second, resulting in over 80% CPU utilization and temporary unresponsiveness. Start/Stop attacks successfully halted the PLC operation within 12 seconds, resulting in a complete process disruption. In MitM scenarios, 98 malicious packets were injected, altering data integrity with a 72% success rate of unauthorized command execution. The artificial intelligence model employed, based on supervised machine learning, achieved a 93.4% detection accuracy across all attack types, with DDoS attacks being the most reliably detected (96.2%) and MitM attacks slightly lower (91.7%). The results highlight that AI-enhanced detection mechanisms not only classify and identify attacks with high precision but also significantly reduce the response time required for mitigation. These findings underscore the importance of integrating AI-driven anomaly detection systems into ICS environments to enhance resilience against evolving cyber threats. Future work will focus on expanding the dataset with additional attack vectors and testing real-time deployment on large-scale industrial networks.

Key Words: Cybersecurity, PLC, Artificial Intelligence, DDoS, Industrial Control Systems



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Radio Frequency Security Risks in Low-Cost IoT Applications: Replay Attack Analysis and Mitigation in the 315/433 MHz Band

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ABSTRACT

The rapid proliferation of low-cost Internet of Things (IoT) devices has introduced significant security challenges, particularly in wireless communication protocols operating on unlicensed frequency bands. This study experimentally investigates the vulnerability of IoT systems using fixed-code transmission in the 315 MHz and 433 MHz frequency bands. A custom-built transmitter-receiver setup, combined with a HackRF One software-defined radio and GNU Radio, was used to capture and replay signals in a controlled environment. Results revealed that replay attacks achieved a 100% success rate, as the receiving devices lacked encryption, authentication, or frequency-hopping mechanisms. Specifically, captured signals—when retransmitted with identical parameters—were consistently accepted as legitimate, enabling full unauthorized control over the target system. During tests, an average of 50 replayed transmissions per trial were performed, all of which bypassed the system without detection. These findings confirm that the absence of rolling codes and lightweight cryptographic schemes in low-cost IoT devices creates exploitable vulnerabilities. Comparative analysis with existing literature highlights similar outcomes in consumer IoT environments such as garage door openers and smart home systems, where simple software-defined radio attacks compromise operational integrity. To address these vulnerabilities, this study proposes the integration of rolling code algorithms, AES-based lightweight encryption, frequency-hopping spread spectrum techniques, and basic authentication protocols. The results emphasize the urgency of adopting “security by design” principles in IoT device development. Without implementing such measures, even inexpensive tools can execute replay attacks that compromise device functionality and user safety.

Key Words: Cybersecurity, PLC, Artificial Intelligence, DDoS, Industrial Control Systems



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Machine Learning-Based Detection of Distributed Denial-of-Service (DDoS) Attacks

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ABSTRACT

The increasing frequency and sophistication of Distributed Denial-of-Service (DDoS) attacks pose critical threats to network availability and security. This study aims to develop an effective classification model for DDoS detection by leveraging supervised machine learning algorithms. The CIC-DDoS2019 dataset was employed, consisting of over 400,000 traffic instances and 78 features, which were reduced to 32 through preprocessing operations, including correlation analysis and feature elimination. Six widely used algorithms—Random Forest, Support Vector Machine (SVM), k-Nearest Neighbors (KNN), XGBoost, LightGBM, and CatBoost—were implemented to classify normal and attack traffic. Model performance was evaluated using Accuracy, Recall, Precision, and F1 Score, supported by confusion matrix analyses. Experimental results revealed that CatBoost achieved the highest performance, with an Accuracy of 98.99%, Recall of 0.8233, Precision of 0.8505, and F1 Score of 0.8341. XGBoost (Accuracy: 98.85%, F1: 0.8184) and KNN (Accuracy: 98.85%, F1: 0.7957) also exhibited strong results, while SVM showed comparatively lower performance (Accuracy: 97.85%, F1: 0.6853). These findings highlight the importance of algorithm selection, as high accuracy values may mask low recall rates that can undermine real-world security effectiveness. Overall, the results demonstrate that supervised machine learning algorithms can effectively detect DDoS attacks, with CatBoost particularly excelling in handling class imbalance and categorical features. The developed models can be integrated into intrusion detection systems to enhance early threat detection and mitigation. This research contributes to the fields of network security, data analytics, artificial intelligence, and cyber defense strategies.

Key Words: Machine Learning, DDoS, Intrusion Detection System (IDS), CatBoost, Cybersecurity.



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A Study on the Performance of Deep Neural Networks for Camouflaged Military Object Detection

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ABSTRACT

Today, computer vision techniques are successfully utilised in numerous critical tasks across various domains, most notably in the defence industry, where object detection applications play a prominent role. In military contexts, the accurate, fast, and reliable detection of strategic assets such as military personnel, tanks, aircraft, and warships through image data is of vital importance, as it enhances both the effectiveness of operational decision-support systems and the overall efficiency of military operations. In this regard, computer vision-based object detection systems have become an indispensable component in modern warfare, contributing significantly to situational awareness and threat analysis processes. However, the detection of heavily camouflaged objects remains a major challenge for existing methods due to factors such as visual similarity, low contrast, and complex backgrounds. Addressing this issue, the present study focuses on the detection of highly camouflaged military objects by employing the comprehensive MHCD2022 [1] dataset, which comprises 3,000 raw images across five distinct classes. In the experimental analyses, the performance of the advanced object detection algorithm YOLOv11 [2] was evaluated on this dataset. The results showed that the model achieved a mean Average Precision (mAP) of 67% and a recall rate of 60.13%, indicating that YOLOv11 outperforms contemporary approaches in the current literature in terms of both consistency and accuracy. Moreover, this study examines the strengths and limitations of the model in detail and aims to contribute to the development of next-generation deep learning models for scenarios involving heavily camouflaged object detection in the military domain.

Key Words: Computer vision, object detection, military object detection, deep neural networks, artificial intelligence

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Sensor Processing Capacity Of The Autonomous Task Computer in Unmanned Aerial Vehicles And Performance Analysis Of Decisions

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ABSTRACT

Unmanned aerial vehicles become more dependent on the capacity to process the sensor data of the task computers due to increasing process loads in autonomous tasks. The lack of timely and reliable sensor data may lead to delays and performance losses in critical functions such as obstacle detection, route planning and emergency response. This study analyzes both experimentally and mathematically the impact of a Raspberry Pi -based task computer connected to the flight control card on sensor processing capacity and decision times. The accelerator, magnetometer and barometer data were evaluated by filtering, time stamping, event -based processing and sensor fusion methods on the task computer. In the free tests performed in the laboratory environment, the time passed from the detection of sensor events to the production production was measured; Delay distributions related to the transaction load, stability during the decision period, processor resource use and temperature changes were analyzed. Within the scope of mathematical analysis, decision delays were modeled with probability distributions and the relationship between transaction load and energy consumption. The findings show that Raspberry Pi keeps decisions at an acceptable level in low and medium -intensity tasks, but delays grow when the transaction load increases and deterioration in sensor accuracy occurs. These results show that the selection of the task computer should be evaluated not only with software flexibility, but also in terms of process capacity, energy efficiency and sensor reliability. In future studies, it is aimed to contribute to the autonomic task performance of UAVs through comparative mathematical analysis of different hardware architecture.

Key Words: Unmanned aerial vehicle, task computer, sensor performance, decision time.

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GEOMETRY



Surfaces with Idempotent Shape Operator Matrix Defined Along a Surface Curve

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ABSTRACT

In this study, we investigate surfaces in differential geometry whose shape operator is idempotent. The mathematical structure of this condition imposes geometric restrictions on the surface, especially concerning its curvatures. The principal aim is to classify such surfaces based on their local curvature properties. The shape operator is a key linear transformation that encodes how a surface bends in space. By imposing the idempotent condition, we are able to derive a system of differential equations from the compatibility of the first and second fundamental forms. These equations provide restrictions on the coefficients of the shape operator matrix, which, in turn, limit the values of the Gaussian curvature K , mean curvature H , and the principal curvatures k_1, k_2 .

Using both analytical techniques and concrete examples, we explore which parametrized surfaces satisfy the idempotency of the shape operator. For each case, curvature values are calculated and used to identify the local geometric nature of the surface. The findings indicate that idempotent shape operators arise in surfaces that are flat in one direction or maintain constant curvature in specific orientations.

This work contributes to the broader goal of surface classifications by linking matrix algebraic properties with intrinsic and extrinsic geometric quantities.

Key Words: Shape operator, Idempotency, Principal curvature, Gaussian curvature, Mean curvature, Differential geometry.



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Semi-invariant Riemannian submersions whose total manifolds have globally conformal Kenmotsu structures

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ABSTRACT

In this talk, we study semi-invariant and its special type, invariant Riemannian submersions whose total manifolds have globally conformal Kenmotsu structures. Using the O'Neill tensors, we first give the equations of covariant derivative for semi-invariant submersions whose total manifolds have globally conformal Kenmotsu structures. In virtue of these equations we give the necessary and sufficient conditions for invariant and anti-invariant distributions of such submersions to be integrable and totally geodesic. After then we give the necessary and sufficient conditions for vertical and horizontal distributions of such submersions to be totally geodesic. Hence, we give the necessary and sufficient condition for the total manifolds of such submersions to be locally product Riemannian manifolds. We also do these examinations for the semi-invariant Riemannian submersions whose total manifolds are globally conformal Kenmotsu and admit horizontal characteristic vector fields. We give an illustrative example for a semi-invariant Riemannian submersion whose total manifold is globally conformal Kenmotsu and admits a horizontal characteristic vector field. Moreover, we give the necessary and sufficient conditions for vertical and horizontal distributions of invariant Riemannian submersions whose total manifolds are globally conformal Kenmotsu to be totally geodesic. We prove that if an invariant Riemannian submersion whose total manifold is globally conformal Kenmotsu admits a horizontal characteristic vector field, then the horizontal distribution of this submersion must be totally geodesic. Finally, we give an example of such an invariant Riemannian submersion.

Key Words: Semi-invariant submersions, conformal structures, Kenmotsu manifolds.



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A Survey on Non-null Slant Ruled Surfaces

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ABSTRACT

In this study, exploiting the definition of non-null slant ruled surface, which is a ruled surface whose Frenet vectors make a constant angle with some fixed directions, in literature, the non-null slant ruled surface acquired by the striction curve of the natural lift curve has been defined with the new perspective in Minkowski 3-space E_1^3 . In order to construct this definition, the correspondence between the subsets of the tangent bundles for pseudo-spheres and E. Study mapping has been considered. This mapping says that there exists an isomorphism between the curves on the tangent bundles of pseudo-spheres and the spacelike or timelike slant ruled surfaces. This mapping has been modified for the non-null slant ruled surfaces generated by the striction curve of the natural lift curve. $\bar{q}$, $\bar{h}$ and $\bar{a}$ spacelike or timelike slant ruled surfaces have been introduced in E_1^3 . Furthermore, some characterizations for $\bar{q}$, $\bar{h}$ and $\bar{a}$ spacelike or timelike slant ruled surfaces generated by the striction curve of the natural lift curve have been denoted. Several significant theorems and remarks have been mentioned in detail. Then, some examples have been given to support the main results. Finally, these results have been discussed and open problems have also been mentioned.

Key Words: Non-null slant ruled surface, E. Study mapping, tangent bundle of pseudo-sphere.

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On Some Geometric Properties of para-Kenmotsu Manifolds

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ABSTRACT

Para-Kenmotsu manifolds constitute an important class of almost paracontact metric manifolds, characterized by their distinctive structure tensors and curvature properties. These manifolds have been the subject of growing interest in differential geometry due to their rich geometric structure and connections with both Riemannian and pseudo-Riemannian geometry. In particular, the study of various curvature tensors such as the Riemann curvature tensor, the Ricci tensor, and generalized curvature tensors including W_0 and W_0^* , provides deeper insights into the intrinsic

geometry of these manifolds.

Understanding the mutual relationships between these curvature tensors not only contributes to the theoretical development of paracontact geometry but also helps in identifying geometric conditions under which the manifold exhibits special properties, such as different types of symmetries, curvature restrictions, and invariance under specific transformations.

In this study, we investigate the interactions among the Riemann, Ricci, W_0 and W_0^* curvature tensors on Para-Kenmotsu manifolds. We aim to determine how these tensors influence one another and to derive conditions under which certain geometric properties emerge. The results obtained enrich the current literature on Para-Kenmotsu manifolds and open new perspectives for further studies in both pure and applied differential geometry.

Key Words: Para-Kenmotsu manifolds, W_0 and W_0^* curvature Tensor, Semi-symmetric manifold.

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On Conformable Slant Ruled Surfaces

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ABSTRACT

In $\mathbb{R}^3$, slant ruled surface, where the Frenet vectors of a ruled surface make a constant angle with some fixed directions in the space, is defined in [1]. In this study, slant ruled surfaces, an important topic in classical differential geometry, are investigated within the scope of conformal analysis techniques. Approximating the tangent at locations where the classical tangent is not easily accessible is one of the main advantages of the conformable derivative. To get around this restriction, alternate tangents can be made by using conformable derivatives. Alternative velocity vectors can be used for physical interpretation because of these alternative (conformable) tangents. Moreover, when it is impossible to specify the tangent plane, the conformable derivative is useful. In terms of expression, the conformable derivative has important implications. It offers information about how a function behaves locally and how it relates to neighboring points. Knowing the conformable derivative helps us better comprehend how a function changes and evolves within its domain.

Utilizing the conformal Frenet frame and its conformable derivative, which defines the kinematic properties of conformal ruled surfaces, conformal slant ruled surfaces are classified as conformable q-, conformable h-, and conformable a- slant ruled surfaces and their geometric and analytical properties are detailed. The results provide a new perspective on the applications of conformal derivatives in surface geometry and offer effective methods for the characterization of Darboux slant surfaces. In the final section of the study, the results obtained by assigning numerical values to α are supported by examples.



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Key Words: Slant ruled surface, conformable derivative, conformable Darboux vector.

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LOCALLY PRODUCT RIEMANNIAN MANIFOLDS WITH CONSTANT CURVATURE

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ABSTRACT

In this talk, we investigate the case of a locally product Riemannian manifold with constant sectional curvature. It is shown that if a locally product Riemannian manifold has constant real sectional curvature, the manifold is flat under certain conditions. Therefore, we examine holomorphic-like sectional curvature and holomorphic-like bi-sectional curvature. However, we show that these cases are not valid. Therefore, a new sectional curvature is introduced and an example is given. For this new locally product Riemannian manifold with constant sectional curvature, the curvature tensor field is obtained in a special form. Next, we investigate whether local Riemannian manifolds with constant new section curvature are Einstein manifolds.

Key Words: Locally product Riemannian manifold, Riemannian curvature tensor field, Constant sectional curvature, Einstein manifolds.

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Generalized Multiplicative Cross Product

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ABSTRACT

A basic operation in vector algebra, especially in three dimensions, is the vector product, which creates a third vector perpendicular to the plane that contains the original vectors. In representation theory, differential geometry, and modern physics, Lie algebras, which are distinguished by a non-associative, skew-symmetric Lie bracket, are essential. This work presents the multiplicative cross product $X \otimes_* Y$ on $\mathbb{R}_*^m$, which is inspired by multiplicative analysis and non-Newtonian calculi. Its basic characteristics are specified and analyzed in detail, including multiplicative orthogonality and multiplicative skew-symmetry. The identification of $(\mathbb{R}_*^m, \otimes_*)$ as a multiplicative Lie algebra, which notably satisfies the multiplicative Jacobi identity, is a key finding of this study. Furthermore, a multiplicative Lie algebra with a well-defined multiplicative Lie bracket is demonstrated to be composed of the set of multiplicative skew-symmetric n -square matrices, A_*^n . These algebraic structures are directly connected and mapped by a multiplicative Lie algebra isomorphism $J : \mathbb{R}_*^m \rightarrow A_*^n$ for $m = \frac{n(n-1)}{2}$. In the advanced context of Lie algebras, these results provide a thorough framework for comprehending multiplicative and generalized vector products.

Key Words: Cross product, Lie algebra, Multiplicative vector product, Multiplicative Lie algebra.

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Application Analysis of Orthogonal Markov Matrices

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ABSTRACT

Orthogonal matrices and Markov matrices play central roles in linear algebra, probability theory, and applied mathematics, they embody fundamentally different structural and dynamical principles. Orthogonal matrices, characterized by orthogonal rows and columns, represent rotations and reflections in Euclidean space while preserving inner products, norms, and geometric structure. In contrast, Markov (stochastic) matrices encode stochastic transitions and preserve the simplex of probability distributions by ensuring that the sum of non-negative row entries reaches unity.

In this study, by examining the intersection of these two classes of matrices, it is shown that simultaneously satisfying the properties of orthogonality and stochasticity imposes stringent constraints. In particular, it is shown that matrices belonging to both groups of matrices act as deterministic Markov processes while also forming a disjoint subgroup of the orthogonal group. This work makes this overlap explicit, explains algebraic and geometric inferences, and positions permutation matrices as the common ground between rigid geometric transformations and probabilistic matrices.

Key Words: Markov matrix, orthogonal matrix

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CONSTRUCTING THE ELLIPSE AND ITS APPLICATIONS IN ANALYTICAL FUZZY PLANE GEOMETRY

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ABSTRACT

In this study, we talked about a method and the details of the method to be used to create the fuzzy ellipse. In the previously studies, some methods for fuzzy parabola are discussed. To define the fuzzy ellipse, it is necessary to modify the method applied for the fuzzy parabola. First, need to get five same points with the same membership grade to create crisp ellipse and the union of crisp ellipses passing through these points will form the fuzzy ellipse. Although it is difficult to determine the points with this property, it is important for constructing the fuzzy ellipse equation. In this study, we determine the points that satisfy this condition and prove the properties required to obtain the fuzzy ellipse to be formed by using these points. In the second part of this study, we have drawn a graph of a fuzzy ellipse and depicted the geometric location of fuzzy points with different membership grades on graph. We have also shown some geometric application on examples. It has been shown that the determinants defined in the calculation of the coefficients of the fuzzy ellipse using the maple program for different points and angles with the examples given, thus different fuzzy ellipses can be obtained. In addition, by investigating the membership of any point to the fuzzy ellipse, if the appropriate conditions are met, the equation of the ellipse passing through this point will be found. Then we can very easily obtain which one is the closest ellipse to the crisp ellipse.

Key Words: Fuzzy ellipse, fuzzy points, same fuzzy points, membership function, α - cuts.



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PHYSICS



The investigation of some new generation alternating shielding materials for the new generation NPP

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ABSTRACT

In this paper, a new structural approximation of materials to strengthen radiation shielding for new generation of nuclear power plants (NPP) are simulated. Thus, radiation shielding properties of different types of materials as vermiculite [1], AC (Activated Carbon) [2] and a few compounds including boron [3,4] are calculated by using MCNP6.2-EXE version of N-PARTICAL Code [5] and then, obtained results are compared to those of currently used ordinary shielding materials like ORNL (Oak Rich National Lab) and MO (Missouri) Concretes, which are known as high performance concretes. This paper also contains theoretical aspects of shielding properties and knowledges about used software system for calculations. After the calculated data were compared to the behaviors of standart shielding materilas, related discussion for their usability in reactor industry is given in the text. At the end, it was seen that the behaviors of such new materials are appropriate and fine for being candidates of new generation shieldings in the aspect of structural approximation.

Key Words: radiation, nuclear power plants, shielding materials

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Radiation Shielding Efficiency of Praseodymium Hexaboride: A Monte Carlo Study

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ABSTRACT

In recent years, the identification, analysis, and evaluation of advanced shielding materials with high absorption capacity for potential applications in nuclear reactors have gained significant attention. In line with the ALARA (As Low As Reasonably Achievable) principle, which aims to minimize personal and public exposure to radiation in nuclear environments. The development of effective shielding technologies is of critical importance to prevent human body from radiation industry. Eventhough, among the next-generation materials, Praseodymium Hexaboride (PrB_6) has attractive interest due to its superior absorption capability of radiation, the studies investigating the potential usage of such innovative materials as the external structures of Nuclear Power Plants (NPPs) are still limited.

In this study, PrB_6 was evaluated alongside boron-based compounds, known for their efficient radiation absorption properties, and vermiculite, a natural shielding material [1-3]. Thus, PrB_6 was modelled as a new generation material of radiation shielding in a computer environment and then the sets of data were analysed. The missing data were completed using MCNP (Monte Carlo N-Particle) and EGSnrc (Electron Gamma Shower) simulation codes. The materials were tested in single, double, and triple-layer configurations to investigate their radiation attenuation performance, and the results were compared with conventional shielding materials currently used in reactors.

Simulation findings were then applied to a selected next-generation NPP model, demonstrating the compatibility of PrB_6 with existing shielding systems as well as its potential advantages. This work is expected to contribute to the development of innovative and effective shielding technologies for nuclear power plants, supporting enhanced safety and sustainability in future nuclear energy applications.

Keywords: Radiation shielding, Praseodymium Hexaboride, Monte Carlo simulation



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Coffee-Ring Effect

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ABSTRACT

The literature on the coffee-ring effect has been reviewed and an analytical solution for the flow velocity inside an evaporating droplet is obtained. The work builds on the pioneering model of Deegan et al., which introduced capillary flow as the principal mechanism behind ring-shaped deposition patterns. Subsequent studies such as Hu and Larson's demonstrated the critical role of Marangoni flows, which can reverse deposition and create central patterns. Malinowski et al. further explored dynamic control of deposition by manipulating surface tension, enabling tunable and versatile pattern formation. In this study, the governing equations for droplet evaporation were revisited under continuity and Navier–Stokes constraints, with boundary conditions extended to include shear stresses. This allowed for improved analytical expressions for radial and vertical velocities within evaporating droplets. Future work involves Monte Carlo simulations to track particle trajectories under these refined velocity fields.

Key Words: Coffee-ring effect, capillary flow, Marangoni flow, evaporation, deposition patterns.

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INVESTIGATING THE MEDIUM EFFECTS ON FULL CHARMONIUM MESON MOLECULES

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ABSTRACT

The experimental observation of exotic hadrons has made significant contributions to both theoretical and experimental studies in high-energy physics. These particles provide valuable insights into the dynamics of quarks and gluons, the nature of strong interactions, and the development of new experimental methods [1]. Although these exotic particles have been observed experimentally, their structures, formation dynamics, and interactions with their environment are not yet fully understood.

In 2020, in the double c production spectrum observed in proton-proton collisions by the LHCb Collaboration, a full charmonium resonance was detected around 6.2 GeV. This resonance was named $T_{c\bar{c}c\bar{c}}(6200)$ [2]. In this study, the spectroscopic properties of the $T_{c\bar{c}c\bar{c}}(6200)$ resonance in a thermal medium were analyzed using the Thermal Quantum Chromodynamics Sum Rules method. The $T_{c\bar{c}c\bar{c}}(6200)$ resonance was modeled as an $\eta_c\eta_c$ molecule with quantum numbers $J^{PC} = 0^{++}$ [3], and in the calculations, non-perturbative contributions up to dimension four were also taken into account.

In this study, the results obtained for the mass and decay constant of the exotic state at low temperatures were found to be consistent with existing theoretical and experimental data in the literature [4-7]. A more recent and accurate fit for the temperature threshold proposed for charmonium states was used in the calculations, and this fit was applied to QCD sum rules for the first time in this study. It was observed that as the temperature approached the critical point, where the transition to quark-gluon plasma begins, the mass of the charmonium molecule started to



decrease, eventually dropping below the $\eta_c\eta_c$ threshold. However, after crossing the critical temperature, the mass increased again, reaching the $\eta_c\eta_c$ threshold around 200 MeV. Similar to the mass, the leptonic decay constant also decreased near the critical temperature, but unlike the mass, the decay constant did not increase with increasing temperature; instead, it remained stable at a value close to that of the η_c . These results suggest that the gluon exchange mechanism begins to relax near the critical temperature and breaks down completely beyond it, and also $\eta_c\eta_c$ particles start to dissolve. Nonetheless, around 200 MeV, the charmonium states recover and regain their structural integrity. These findings align with the behavior proposed in the literature for heavy quarkonium states [8] and have been theoretically demonstrated for the first time in this study.

Key Words: Thermal QCD sum rules, Exotic mesons, Full charmonium molecule state, Mass and decay constants.

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TOPOLOGY



A Research on Operators Γ^{ψ_Γ} and ψ_Γ^Γ

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ABSTRACT

In general topology, the concept of the ideal [3, 5] has been studied since 1944. There are numerous studies related to the concept of the ideal in the literature. Especially, Jankovič and Hamlett have conducted in-depth research on ideal topological spaces in [2]. They obtained new topologies from old via ideals [2]. Simultaneously, there are many articles about operators which obtained by utilizing ideal in the literature. The most notable among these are the local function [3] and the operator ψ [6]. The operator $*\psi$ [7] and the operator ψ^* [8] are also defined by using the local function and the operator ψ . On the other hand, Al-Omari and Noiri defined the operator Γ , called the local closure function, and the set operator ψ_Γ [1] in 2013. In addition to these studies, Yalaz and Keskin Kaymakçı introduced the $\Gamma - I$ –open sets via local closure function in [4].

In this study, we present new type of set operators Γ^{ψ_Γ} and ψ_Γ^Γ with the help of the local closure function and the operator ψ_Γ in ideal topological spaces. Besides, we explore the basic properties of these set operators in detail and obtain some important findings. Moreover, we present some counterexamples that are related to our findings. We also examine the features of these operators in some special ideals as well. In this context, we conclude that these set operators depend on both the ideal and the topology in ideal topological spaces.

Additionally, by using the operator Γ^{ψ_Γ} , we present the notion of new set called Γ^{ψ_Γ} –set. We analyze the relationships between Γ^{ψ_Γ} –sets and various sets in the literature in general and under certain specific conditions.

Key Words: Local closure function, operator ψ_Γ , Γ^{ψ_Γ} –set.



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Coverings of Hom-Lie Crossed Modules

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ABSTRACT

Crossed modules on groups were defined by Whitehead [1]. After that, they are defined on many algebraic structures such as algebra, commutative algebra, Lie algebra, Leibniz algebra, Lie-Rinehart algebra and their properties, and their relationship with other structures has been investigated [2, 3]. Recently, crossed modules over Hom-Lie algebras were introduced by Shen and Chen in [4] in order to reveal relationship between Hom-Lie 2-algebras and crossed modules on Hom-Lie algebras. After then, many authors studied this topic. Especially, Casas and Garcia-Martinez investigated crossed modules of Hom-Lie algebras in [5], and they studied on low dimensional cohomology groups of Hom-Lie algebra and their relation with crossed modules. Also, they defined cat^1 -Hom-Lie algebras and showed natural equivalence between category of cat^1 -Hom-Lie algebras and category of crossed modules over Hom-Lie algebras.

Covering for crossed modules was first defined by Brown and Mucuk in [6] using the equivalence of crossed modules and group-groupoids. The concept of coverings for groupoids were obtained using fundamental groupoid functor (π_1) by Brown, Danesh-Naruie and Hardy [7]. In the same study, they defined coverings for internal groupoids in any category with pullback structures. With similar thinking, in this study, we will obtain coverings of Hom-Lie crossed modules and give some their properties.

Key Words: Hom-Lie algebra, crossed module, covering.

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A note on quasi-metrizable spaces

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ABSTRACT

Let X be a non-empty set and A, B two subsets of $X \times X$. By definition,

$$A^{-1} = \{(a, b) : (b, a) \in A\}, \quad t \in [0, T],$$

and

$$A \circ B = \{(a, c) : \text{there exists } b \in X \text{ such that } (a, b) \in B \text{ and } (b, c) \in A\}.$$

A filter $\mathcal{U}_q$ on $X \times X$ is called a quasi-uniformity if each element of $\mathcal{U}_q$ contains the diagonal set and for each $W \in \mathcal{U}_q$ there exists $V \in \mathcal{U}_q$ satisfying $V \circ V \subseteq W$.

It is well known that a quasi-uniformity generates a topology. The converse, that is, there is a quasi-uniformity for a given topology compatible with the topology, was first proved by Krishnan [2], then Császár proved it with the structure he defined [1]. But the direct topological proof was given by Pervin [3].

A subcollection of a given quasi-uniformity is a basis for the quasi-uniformity if each element of the quasi-uniformity includes at least one element of the subcollection in question. A function $d : X \times X \rightarrow [0, \infty[$ is said to be a quasi-metric if $d(x, x) = 0$ and $d(x, z) \leq d(x, y) + d(y, z)$ ($x, y, z \in X$). In this case, X is said to be quasi-metrizable if the family $(B_\lambda(a))_{\lambda > 0}$ is a local basis at each $a \in X$ where $B_\lambda(a)$ is the set containing all points $b \in X$ satisfying $d(a, b) < \lambda$.

In this talk, we discuss the conditions to obtain a quasi-metrizable topological space.

Key Words: quasi-uniformity, quasi-metric



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A note on global classical solutions to a Cauchy problem

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ABSTRACT

Let $F : [0, T] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a set-valued map with nonempty values. In this paper, we study the existence of global classical solutions to the following problem, called a Cauchy problem such that $x'(0) = y_0$, for every $x_0 \in \mathbb{R}^n$ and $y_0 \in F(0, x_0)$,

$$x'(t) \in F(t, x(t)), \quad t \in [0, T],$$

$$x(0) = x_0, \quad x'(0) = y_0.$$

The existence of local (or global) classical solutions in the nonconvex (or convex) case for this problem, under the assumption that each set $F(t, x)$ is uniformly locally connected (or totally disconnected), was studied in many papers (see, e.g. [1], [2]). For this purpose, we make use of some results related to the existence of Caratheodory type solutions under different assumptions.

Key Words: differential inclusion, existence, classical solution

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Some Properties of Metric in Digital Topology

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ABSTRACT

The aim of this study is consider some properties of a digital image. A digital image X is a subset of $\mathbb{Z}^n$ for some natural number n together with an adjacency relation κ inherit from $\mathbb{Z}^n$ and represented by (X, κ) . We analyze digital images by using the shortest digital simple path metric which is denoted by d^k . We use digital topological concepts such as adjacency relation, the r -thickening of the subsets of a connected digital image and *digital Hausdorff distance* to explore the properties of digital images with the shortest digital simple path metric. The r -thickening of a subset is defined by taking the union of neighborhoods with radius r of all points in subset. The *digital Hausdorff distance* between two noempty subsets A and B of κ -connected digital image X is defined minimum r such that the r -thickening of each subset will contain another.

In this study we define distance of two subsets of a connected digital image via the shortest simple path metric. We examined digital topological versions of metric-related features using the shortest simple path metric. Applications on various subsets of digital images are explored. We give examples showing that when the adjacency relation defined on a digital image changes, the distance between the points taken on the image changes.

Key Words: Digital topology, digital image, Hausdorff metric.

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Banach and Kannan Type Contracting Mappings for Box Metric in Digital Images

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ABSTRACT

The notion of digital topology is a field that uses geometric and algebraic topology to examine the geometric and topological properties of digital images. Digital topology has applications used in computing areas such as image processing and computer graphics. Digital topology is a field of mathematics that develops by using the topological properties of objects, and more by using the characteristics of two and three-dimensional digital images.

On the other hand, fixed point theory is one of the important and developing areas of mathematics. One of the most fundamental theorems in fixed point theory is Banach fixed point theorem. The reason why this theorem is important is that it has a wide range of applications and has created fundamentals for proving different fixed point theorems. One of the important theorems proven after the Banach fixed point theorem is Kannan fixed point theorem. Kannan fixed point theorem is a theorem that is still being studied in various metric spaces and its applications are presented. The applications of fixed point theory are used in many scientific areas.

In this study, firstly, preliminaries about digital topology, fixed point theory and box metric are given. The properties that distinguish digital metric spaces from metric spaces are examined. The most prominent of these properties is that the ε is taken as 1 because it is studying on a finite subset of $\mathbb{Z}^n$. This property has played an important role in defining basic concepts such as convergence sequence and Cauchy sequence in digital metric spaces. In digital images, the Banach fixed point theory is then proved by defining a special metric called box metric, considering this metric in digital images with κ -adjacency relation and using the properties of digital metric space. Then, Kannan fixed point theorem is given by expressing Kannan type digital contraction mapping in digital box metric spaces.

Key Words: Digital box metric, Banach fixed point theory, Kannan type digital contraction mapping.

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FUNDAMENTALS OF MATHEMATICS AND MATHEMATICS LOGIC



Incorporating Expert Confidence Levels into Fuzzy Multi-Criteria Decision-Making under Uncertainty

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ABSTRACT

Decision-making in complex environments often involves uncertainty, incomplete data, and varying levels of expert reliability. Traditional fuzzy set approaches provide useful tools for modeling imprecise information but typically overlook the impact of decision-makers' confidence levels. This study presents a fuzzy multi-criteria decision-making (MCDM) framework that explicitly integrates expert confidence into the evaluation and aggregation process. The method is developed within the Picture Fuzzy Set (PFS) environment, which captures membership, non-membership, and neutrality degrees simultaneously, offering a richer representation of uncertain judgments. A new class of power-based aggregation operators is proposed to combine expert assessments while weighting them according to individual confidence levels. The mathematical validity of the operators is established by proving essential properties such as idempotency, boundedness, and monotonicity. To demonstrate practical applicability, the framework is applied to a supplier selection problem, showing that incorporating confidence levels leads to more robust and consistent rankings compared to existing models. This approach provides a flexible decision-support tool suitable for various domains, including industry, healthcare, and environmental management, where expert reliability plays a critical role in final outcomes.

Key Words: Multi-criteria decision-making, Picture fuzzy set, Food industry.

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MATHEMATICS EDUCATION



INVESTIGATION OF MATHEMATICAL LITERACY ACTIVITIES PREPARED BY PRESERVICE TEACHERS TO ELIMINATE MISCONCEPTIONS

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ABSTRACT

This study, which is supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK), highlight the design and evaluation of mathematical literacy-based instructional activities developed by preservice teachers aimed at remedying misconceptions in the 7th-grade algebra curriculum. While most studies in mathematics education focus on identifying misconceptions (Türkdoğan et al., 2015), this research makes an original contribution by emphasizing the development of pedagogical interventions to resolve the mentioned misconceptions.

In compliance with the conference requirements, both the abstract and presentation slides will be prepared in English. However, permission is respectfully requested to deliver the presentation in Turkish, as I believe it will allow me to communicate result more effectively. All accompanying written and visual materials will be presented in English to ensure accessibility for the international audience.

This study aims to investigate the activities designed to eliminate misconceptions in the 7th-grade algebra learning area. Therefore one-question test was prepared so as to enable preservice mathematics teachers to predict possible misconceptions in the algebra curriculum. The test presented algebra learning outcomes to the participants, followed by the question: "What possible misconceptions might students experience?" They were then asked to design a mathematical literacy activity aimed at addressing one of the misconceptions they had identified.

During the project, participants were given 7th-grade algebra learning outcomes and asked to write possible misconceptions. Books containing activities related to mathematical literacy and secondary mathematics were provided. Participants had one month to prepare mathematical literacy activities aimed at eliminating identified misconceptions accordingly. The suitability of these activities to the 7th-grade algebra



topics and mathematical literacy was evaluated. The activities were eventually scored based on Özgen's (2021) checklist, and their content was examined for alignment with the identified misconceptions.

The data analyzed by qualitative methods. The appropriateness of the mathematical literacy activities created by preservice teachers was evaluated according to Özgen's (2021) checklist, which includes criteria such as content, level, process, context, and structure. Content analysis was conducted to determine the extent to which the relevant concept was addressed. To ensure validity, two researchers coded the data independently.

The findings of the study indicate that pre-service teachers were successful in identifying misconceptions related to 7th-grade algebra learning objectives and in designing mathematics literacy activities aimed at addressing these misconceptions.

Key Words: mathematical literacy, misconceptions, algebra education, preservice teachers, content analysis

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Examining the Norms of the Problem Solving Process within the Framework of Practical Rationality

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ABSTRACT

This research examines the norms that emerge during problem-solving processes within the framework of the Theory of Practical Rationality. The study aims to uncover the underlying norms behind the behaviors and decisions exhibited by participants during their problem-solving actions. The problem-solving process is based on Polya's (1945) phased approach, systematically analyzing how participants understand the problem, devise a plan, execute the plan, and evaluate the solution process.

The Theory of Practical Rationality, developed by Herbst and Chazan (2011), allows for understanding how rational decisions are made and how they guide solution processes in mathematical situations during mathematics education. According to this theory, individuals' actions are guided by norms specific to their social environment and professional practices. Therefore, problem-solving actions are not merely cognitive processes but are interwoven with these norms.

The study included mathematics teachers with varying professional experiences and academics working in the field of mathematics education. Two different problem contexts were used, and the participants' solution processes were collected using a qualitative method, namely focus group interviews (Krueger and Casey, 2015). Focus group interviews were chosen as an appropriate method to deeply understand participants' thought processes, strategy preferences, and the dilemmas they faced during the solution path. The data obtained from four different focus group interviews, each corresponding to a step in the problem-solving process, were analyzed using content analysis to determine the norms related to the problem-solving processes (Miles, Huberman and Saldaña, 2019). These interviews allowed participants to collectively experience the problem-solving processes in two different complex and open-ended problem contexts. The analyses enabled the systematic presentation of the rational approaches exhibited by the participants in their solution processes.

As a result of the analyses, the main norms displayed by the participants during their problem-solving processes were determined. These norms were found to be closely related to Polya's problem-solving steps. For example, in the understanding the problem step, participants exhibited norms of checking prior knowledge and analyzing the given information in the problem. In the devising a plan phase, norms such as simplifying the problem, leveraging similar solved problems, and supporting the use of different strategies came to the forefront. In the executing the plan and evaluating the solution steps, norms like asking for explanations of solutions, self-evaluation, peer evaluation, and posing a problem that takes the problem-solving process a step further were observed. Furthermore, as a common decision-making



norm for the participants, the practice of preparing before problem-solving was found to be significant.

The findings of the research demonstrated which norms were adopted by the participants in their problem-solving processes and how these norms shaped their solution strategies. In addition, the evaluations made within the framework of the Theory of Practical Rationality contributed to understanding the role of norms in instructional processes and teachers' pedagogical decisions. This study contributes to the mathematics education literature by determining the norms related to the problem-solving process within the scope of the Theory of Practical Rationality and the applicability of this approach in education. Furthermore, this work shows that the problem-solving process is not just a series of cognitive skills but also a product of professional practices and social interactions. These findings offer important implications for the design of training programs for the professional development of mathematics teachers and the teaching of problem-solving skills, suggesting that practical rationality norms should be considered in addition to cognitive processes.

Key Words: Practical Rationality, Norms, Problem Solving Process.

* This study was developed from a section of the doctoral dissertation title "Examining Preservice Mathematics Teachers' Practical Rationality of Problem Solving Process" prepared by the second author under the supervision of the first author at the Marmara University Institute of Educational Sciences.

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An Examination of Seventh-Grade Students' Symbol Sense Through Technology-Supported Tasks¹

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ABSTRACT

Nowadays, technology has become an essential component of algebra instruction. Through technology-supported learning environments and dynamic software, students are able to concretize abstract algebraic concepts and develop a deeper understanding. Arcavi [1] introduced symbol sense as an often-overlooked aspect in the field of algebra learning and identified its components. In his study, Arcavi [2] defined the components of symbol sense as follows: Friendliness with symbols, an ability to manipulate and also to ‘read through’ symbolic expressions, successfully engineer symbolic relationships, select one possible symbolic representation for a problem, check for the symbol meanings during the implementation, and play different roles in different contexts.

The aim of this study is to examine the symbol sense of seventh-grade students in technology-supported tasks within the domain of algebra learning. A teaching experiment design, as part of qualitative research, was employed. The study group consisted of four seventh-grade students with heterogeneous academic achievement levels and social communication skills, including one student with high achievement, two with medium achievement, and one with low achievement in mathematics. The study was conducted as a group study.

For the technology-supported learning environment, the “Amplify Classroom” application was chosen. Within this application, 11 tasks were designed that comprehensively addressed the components of symbol sense. Before starting to use

¹ This study was produced from the doctoral thesis conducted by the first author under the supervision of the second author.



Amplify Classroom, its dynamic features were introduced to the students and guidance was provided on how to use the tools.

In this study, a part of the task entitled "Balance" was presented. Data analysis was carried out using the MAXQDA qualitative data analysis program. It was observed that students with high and medium academic achievement demonstrated more behaviors associated with symbol sense components and, moreover, were able to exhibit behaviors belonging to multiple components simultaneously. In contrast, the student with low academic achievement displayed fewer symbol sense behaviors compared to peers, and showed limited relational behaviors involving multiple symbol sense components.

Within the scope of the task, it was observed that students most frequently engaged in behaviors related to the ability to manipulate and also to 'read through' symbolic expressions, as well as check for the symbol meanings during the implementation. Furthermore, these two components were found to operate in close connection with one another. It was also observed that the technology-supported learning environment contributed to the emergence of students' behaviors associated with successfully engineering symbolic relationships. Additionally, through their behaviors related to check for the symbol meanings during the implementation and play different roles in different contexts, students were able to question and make sense of zero and negative numbers using the dynamic balance model. One of the most significant findings of the study was that students solved one-sided equations on the balance model by employing the ability to manipulate and also to 'read through' symbolic expressions. Future studies may investigate how the development of symbol sense differs across various grade levels in technology-supported learning environments.

Key Words: Symbol sense, algebra, technology-supported learning environment.



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Modeling Problem Solving Process Interaction Analysis: A Discourse Study on the Parking Lot Design Problem

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ABSTRACT

This study aims to analyze the discourse moves exhibited by pre-service mathematics teachers while solving a mathematical modeling problem in a collaborative setting. The participants consisted of three third-year students from the elementary mathematics education program at a public university. They worked as a group to solve the "Parking Lot Design" modeling problem during a 90-minute session. All verbal interactions were audio-recorded and transcribed. The data were analyzed using discourse analysis across three dimensions: discourse channel, discourse level, and discourse type.

A total of 985 discourse moves were identified. The findings revealed that interpersonal discourse was dominant (82%), although personal discourse (18%) also played a notable role. Non-object level discourse (58%) was more prevalent than object-level discourse (42%), indicating a strong focus on process organization and group coordination beyond pure mathematics. In terms of discourse types, information sharing was the most frequent (41%), followed by questioning (22%) and confirmation (19%), which supports earlier findings that collaborative learning improves problem-solving through peer interaction and support (Klang et al., 2021; Purba & Ramadhani, 2020). Participants adopted distinct roles: Participant 1 was primarily an information provider, Participant 2 acted as a supporter, and Participant 3 as a questioner and critic, reinforcing the importance of role diversity for productive mathematical discourse (Syafri et al., 2020).

Overall, this study reinforces the critical role of discourse in mathematical problem-solving processes and underscores the value of structured collaboration. It



further suggests that integrating discourse-rich modeling tasks into teacher education programs can enhance pre-service teachers' mathematical reasoning, communication, and reflective thinking skills (Robinson, 2020; Biza, 2017). These insights contribute to the growing body of research advocating for discourse-based pedagogies within mathematics education.

Key Words: Mathematics education, discourse analysis, modeling, pre-service teachers, group interaction

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Analysis Of Technological Pedagogical Content Knowledge Studies in The Field of Mathematics Education

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ABSTRACT

The professional development of teachers, their ability to transfer qualified knowledge to students, and the creation of effective learning environments require them to possess adequate competence in their respective fields (Hiebert et al., 2003). Particularly in mathematics teaching, it is insufficient for teachers to know the fundamental concepts of mathematics; they are also expected to possess pedagogical knowledge regarding how to teach these concepts to students (Karagöz Akar, 2010). Indeed, the effective use of technology in educational processes enhances students' academic achievement and renders learning processes more efficient (Peker, 1985). Therefore, examining the trends of studies conducted in this field is crucial for identifying the existing gaps in the literature and providing a foundation to guide future research.

This study examined research conducted between 2020 and 2024 within the scope of Technological Pedagogical Content Knowledge (TPACK) in mathematics education. The study adopted a systematic literature review method, and the databases ULAKBİM, ScienceDirect, Wiley Online Library, Taylor & Francis, Springer Link, and Web of Science were searched. Specific criteria were applied to the studies included in the research: they had to be published in Turkish or English, contain at least one of the concepts "TPACK," "TPCK," or "technological pedagogical content knowledge" along with the term "mathematics," be in article or conference paper format, and be available in open-access full text. Seventy-four studies meeting these criteria were included in the research and analyzed regarding cognitive domain themes, affective domain themes, and mathematical subfields.

The cognitive domain constitutes the learning dimension that involves mental processes such as acquiring knowledge, comprehension, analysis, synthesis, and evaluation (Bloom et al., 1956). The affective domain, on the other hand, encompasses students' feelings, attitudes, interests, values, and motivations toward learning (Bloom et al., 1956). This domain determines students' engagement, willingness to learn, and the emotional connection they establish with the process. Accordingly, considering cognitive and affective domains together contributes to developing a holistic understanding in mathematics education research.

The study's findings indicate that research on cognitive domain themes is more prevalent in the literature than studies focusing on affective domain themes. Within the cognitive domain, the theme of "knowledge" emerged as particularly prominent. Studies addressing themes such as development, competence, and experience were also identified. In affective domain research, however, the theme of "attitude" was



examined more extensively. Furthermore, themes such as self-efficacy, perception, and expectation were also relatively more frequently investigated within the affective domain. Concerning mathematical content, the studies were predominantly concentrated in the geometry subfield, while algebra emerged as one of the most frequently investigated areas. This indicates a concentration of research in specific domains within the mathematics education literature and points to the need for further studies in other mathematical subfields.

(This study was derived from the master's thesis of the first author, Şeyma BARBAROS, entitled "Analysis of Technological Pedagogical Content Knowledge Studies in the Field of Mathematics Education.")

Key Words: Mathematics Education, Technological Pedagogical Content Knowledge, Systematic Review

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Contemplation in Mathematics

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ABSTRACT

The aim of this study is to demonstrate that mathematics education is not limited to teaching operations and formulas; rather, it is also a means of leading students to contemplation on the order, balance, and measurement of the universe. Mathematics is one of the most powerful tools of human reasoning, yet in the educational process it is often approached only in its technical dimension, while its aspect of wisdom and contemplation is neglected. This article examines the contemplative dimension of mathematical concepts (such as sets, numbers, functions, limits, and analytical geometry and so on) and shows that these can contribute not only to students' intellectual development but also to the expansion of their spiritual perspectives. Considering the emphasis of the belief on reason and reflection, mathematics education is not merely a technical training but also an invitation to contemplation and understanding of divine wisdom.

Mathematics education in modern curricula is often focused on problem solving and formula memorization. However, mathematical thinking is not only an intellectual activity but also a way of deeply understanding existence and perceiving the measure and harmony in its functioning. Therefore, mathematics education should be regarded not simply as the acquisition of procedural skills but also as a process of reasoning, comprehension of order, and grasp of wisdom.

Fundamental subjects of mathematics education —such as sets, numbers, functions, limits, analytic geometry, and probability— can all be linked to contemplation. The concept of sets is not merely limited to the teaching of collections of elements; it also points to the classification and organization inherent in the universe. The existence of millions of different living species reflects a profound order, which is not a product of randomness but rather an indication of a powerful Creator's wisdom.



The contemplation of numbers can be approached on two levels. On the one hand, immense numbers – such as the stars in galaxies, distances measured in light years, or the trillions of cells in the human body – remind us of the greatness of divine power. On the other hand, the study of minute realities – such as atoms, protons, or the billions of base pairs in DNA – points to the depth of divine knowledge and wisdom. Functions, which assign an output to each input, symbolize purposeful and organized processes. Similarly, the functions of human organs are not random but directed toward specific goals, indicating the existence of a power that programs and sustains them.

The concept of limits introduces students to both the infinitesimal (approaching zero) and the infinite (approaching infinity). This mathematical perspective encourages reflection upon astronomical distances as well as subatomic structures, leading to awe and contemplation both in the vast and the minute. Likewise, symmetry in nature –such as that observed in flowers, snowflakes, or the balance of the human face – together with the geometric ornaments of mosques and the harmony in Islamic calligraphy, reflects a perfection that cannot be attributed to chance. This harmony resonates with the Qur’anic notions of measure (mizan) and order (intizam).

All these examples demonstrate that mathematics education is not merely a field for acquiring technical knowledge but also a powerful tool for contemplation. In light of the Qur’an’s frequent calls to reflection, mathematics courses may contribute to both the cognitive and spiritual development of students. If the universe is considered a book (Kitab al-Kabir), mathematics is a science that helps humankind to read this book and to recognize its Author. For this reason, integrating contemplation more consciously into mathematics education can enable students to harmonize scientific reasoning with spiritual understanding.

Key Words: Mathematics education, contemplation, sets, numbers, functions, limits, analytical geometry, probability universe, symmetry

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The Development of Algebraic Thinking in the Instructional Interventions: A Systematic Review

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ABSTRACT

Algebraic thinking is a fundamental component of mathematics education and plays a central role in developing students' critical thinking, problem-solving, generalization, and mathematical reasoning skills. The PISA (2022) results indicate a significant decline in students' mathematical abilities worldwide, with particular deficiencies observed in algebraic proficiency (OECD, 2023). According to PISA data, there is a strong relationship between students' mathematics achievement and their algebraic thinking skills (Yanto et al., 2022). Students with higher algebraic abilities tend to be more inclined toward critical thinking, reasoning, and problem-solving, which are central objectives of mathematics education (Žakelj et al., 2024). Nevertheless, algebra remains one of the most challenging subjects for students (Carraher et al., 2006; Kaput, 1999; Kieran, 1992). Traditional algebra instruction often reduces learning to symbols and procedures, hindering meaningful understanding and directing students toward rote-based problems. This highlights the need for innovative approaches in mathematics education that support the development of algebraic thinking. Consequently, research studies have been conducted focusing on instructional interventions aimed at enhancing algebraic thinking.

To identify trends, detect gaps, and provide guidance for the field, these studies need to be examined in a systematic and comprehensive framework. The purpose of this systematic review is to holistically analyze research studies that have investigated instructional interventions designed to develop students' algebraic thinking skills. Systematically reviewing existing research not only reveals trends and gaps in the field but also provides direction for future studies. The review analyzed the selected studies in terms of: (i) instructional interventions and teaching approaches aimed at developing algebraic thinking, (ii) tools used (manipulatives,



digital environments, software), (iii) sub-dimensions of algebraic thinking (equations, variables, functional thinking, generalization, transitions between representations), (iv) participant profiles and age groups, (v) methodological approaches (quantitative, qualitative, mixed methods, design-based approaches), (vi) data collection instruments, (vii) findings obtained, and (viii) researchers' recommendations.

The systematic review was conducted following the PRISMA model. The Web of Science database was searched using the following terms: Topic = ("algebraic thinking" OR "algebraic reasoning" OR "algebra skills") AND ("teaching" OR "instruction" OR "intervention" OR "educational practice" OR "instructional approach") AND ("students" OR "learners" OR "pupils"). The search yielded a total of 228 studies. Of these, 153 studies were within the field of education and educational research, and 116 were identified as articles. Based on the inclusion criteria (excluding review articles, studies outside the field, studies relying solely on teacher perspectives, and studies not directly focusing on algebraic thinking), 62 studies were included in the analysis. As a result, this review will present research trends, content orientations, and methodological gaps related to algebraic thinking. The study aims to provide a systematic framework for interventions that support the development of algebraic thinking and serve as a reference for future research.

Key Words: Algebraic thinking, instructional interventions, systematic review.

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AN EXAMINATION OF DISCURSIVE MOVES IN HORIZONTAL AND VERTICAL MATHEMATIZATION PROCESSES WITHIN THE REALISTIC MATHEMATICS EDUCATION APPROACH

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ABSTRACT

This study explores the use of discursive moves in horizontal and vertical mathematization processes within a Realistic Mathematics Education (RME) environment. Focusing on the Pythagorean Theorem, it examines how teacher discourse supports students in transitioning from contextual modeling to mathematical generalization. The study was conducted in a sixth-grade classroom with four selected students working on a real-life problem inspired by the Flatiron Building. The data were collected through classroom observations, audio recordings, worksheets, and student interviews.

A qualitative case study design was used, and teacher-student dialogues were analyzed using the SHKK (Discursive Moves Coding Catalogue). During horizontal mathematization, the teacher predominantly employed informative (35.1%), communicative (35.1%), and comparison-based (24.3%) moves to guide students in connecting real-world context with mathematical representations. In vertical mathematization, communicative moves (37.5%) remained dominant, followed by justification, naming, and monitoring strategies (each 12.5%), which supported abstraction and formalization of the concept.

The findings indicate that discursive strategies evolve across mathematization phases and play a crucial role in shaping students' mathematical understanding. Purposeful teacher discourse facilitates the development of reasoning skills and fosters meaningful mathematical learning experiences in RME-based classrooms.

Key Words: Realistic Mathematics Education, mathematization, discursive moves



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A TEACHING ACTIVITY BASED ON A HYPOTHETICAL LEARNING TRAJECTORY: MODELING OF IDENTITIES

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ABSTRACT

This study aims to investigate the learning processes of eighth-grade students regarding the modeling of algebraic identities through a teaching activity based on a Hypothetical Learning Trajectory (HLT). The research problem guiding this study is: "How do eighth-grade students' learning processes develop when they model identities within a teaching activity based on a Hypothetical Learning Trajectory?"

The participant of this qualitative case study was one eighth-grade student enrolled in a public school in Samsun during the 2023–2024 academic year. The school was selected through convenience sampling, and the student was chosen via criterion sampling. Prior to the intervention, a readiness test was administered to identify prerequisite knowledge. Afterwards, an HLT was designed and implemented step by step with the student. Data were collected through the readiness test, activity sheets, individual worksheets, observations, interviews, and a final evaluation test. Expert opinions were obtained to ensure the validity of the instruments. Data were analyzed using content analysis, a qualitative method.

Findings revealed that the student achieved the targeted learning outcomes through the HLT-based instruction and did not encounter major obstacles, except for a difficulty in recognizing algebraic expressions as common factors. This challenge was addressed with additional activities, and the HLT was revised accordingly. The process showed that the designed trajectory effectively supported the student's academic progress in modeling identities.

The study highlights the potential of HLTs to guide instruction in algebra by identifying key milestones, predicting possible misconceptions, and supporting meaningful learning. Using algebra tiles and visual modeling facilitated the student's conceptual understanding and strengthened the connection between visual and algebraic representations. These results suggest that HLT-based instruction can contribute to designing effective learning environments in mathematics education and provide valuable insights for teachers to support students' developmental levels.



Key Words: Hypothetical Learning Trajectory, Algebraic Identities, Modeling, Mathematics Education

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A National Model Proposal for Metacognition-Based Mathematics Teaching: ÜSPİDMAT

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ABSTRACT

Metacognition, understood as the ability to plan, monitor, and evaluate one's own cognitive processes, has emerged as a key factor in mathematics learning and teaching. Beyond the possession of knowledge and strategies, successful problem solving requires awareness of how, when, and why to use them. Research highlights that metacognitive strategies not only enhance problem-solving performance but also reduce mathematics anxiety, promote independent learning, and support long-term academic achievement.

Building on international approaches such as IMPROVE (Israel), CRIME (United Kingdom), and MURDER (United States), this study introduces ÜSPİDMAT (Mathematics Teaching Based on Metacognitive Planning, Monitoring, and Evaluation) as a nationally grounded model tailored to the Turkish educational context. ÜSPİDMAT provides structured guidance for teachers to embed metacognitive awareness in classroom practices, thereby encouraging strategic thinking and deeper mathematical understanding.

This research constitutes a significant component of a larger national project that aims to integrate metacognition into mathematics teaching in Turkey. By situating ÜSPİDMAT within both theoretical foundations and practical classroom needs, the project aspires to create a sustainable model that bridges global research with local implementation. Consistent with the findings of Okumuş and Öztürk (2024), who emphasized the effectiveness of metacognition-based mathematics teaching, the present proposal positions ÜSPİDMAT as a comprehensive framework for advancing mathematics education at a national level.



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Key Words: metacognition, mathematics, keyword three.

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